

Metric Ellipses in Minkowski Planes

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ABSTRACT

An ellipse in \mathbb{R}^2 can be defined as the locus of points for which the sum of the Euclidean distances from the two foci is constant. In this paper we will look at the sets that are obtained by considering in the above definition distances induced by arbitrary norms.

Let $X = (\mathbb{R}^2, \|\cdot\|)$. For $x \in X$, $\|x\| = 1$, and $c > 2$ next figure shows the metric ellipse $E(x, c) = \{y \in X : \|x + y\| + \|x - y\| = c\}$ for different norms:

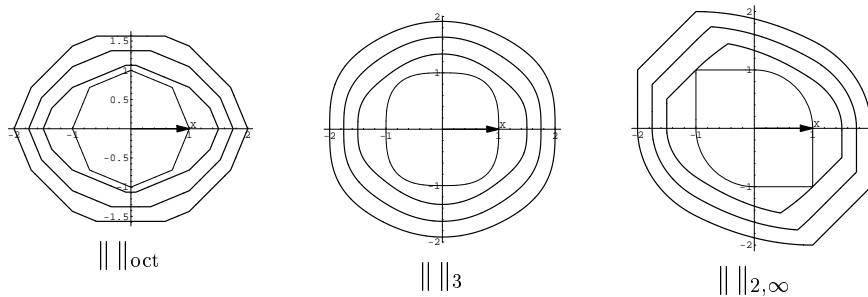


Figure 1: Metric ellipses $E((1, 0), c)$ for $c = 3, 3.5, 4$ and several norms.

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