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Filtering of Signals Transmitted in Multichannel from Chandrasekhar and Riccati Recursions*

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1. Introduction

For a long time, the recursive algorithms proposed as a solution of the least mean-squared error (LMSE) linear estimation problem of signals in stochastic systems have been expressed by means of Riccati-type difference or differential equations. Nevertheless, the interest of finding fast algorithms has led many authors to replace those Riccati-type equations by a set of Chandrasekhar-type ones. By using this kind of equations, a reduction of the number of operations at each iteration of the algorithm and so, a decrease in the computation time, is achieved.

The first authors who used the Chandrasekhar-type equations to solve the LMSE linear estimation problem in discrete-time systems were Morf et al. [7], in the mid-seventies. For time-invariant systems, these authors obtained a new algorithm which, by reducing the number of difference equations contained in it, improved computationally the celebrated Kalman filter.

From this work, there have been many authors who have proposed Chandrasekhar-type recursive algorithms to solve different estimation problems, applying those algorithms to different real situations (air pollution

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over an urban area ([1]), image restoration ([6]), digital communication ([16]), etc.). We will especially mention the results obtained by Sayed and Kailath [15], who generalized the Chandrasekhar-type algorithm proposed in [7] to a class of time-variant state-space models; this work constitutes the basis of subsequent researches in the multichannel and multidimensional adaptive filtering context (see e.g. [3] and [14]).

On the other hand, assuming that the state space model is not completely known but only using covariance information, Friedlander et al. [2] derived Chandrasekhar one-step prediction equations in linear discrete-time stochastic systems. Recently, by using also covariance information, Nakamori [9] has proposed a Chandrasekhar-type filtering algorithm for a wide-sense stationary scalar signal in continuous-time systems. The same problem has been analyzed in discrete-time systems by Nakamori et al. [13] assuming uncertain observations, who have also proposed to solve it an algorithm based on Chandrasekhar-type equations. Our aim, in this paper, is to generalize the study approached in [13] to the case of uncertain observations with non-independent uncertainty, perturbed by white and coloured additive noises.

Systems with uncertain observations have been widely studied since many practical situations (as in communication theory, control systems, robotics, aerospace navigation, vehicular traffic theory, etc.) can be modelled by this kind of systems. Its main characteristic is that the signal is not always present in the observations but its presence in them is subject to a probability. This property is reflected in the observation equation by means of a multiplicative noise, described by a sequence of Bernoulli random variables.

The LMSE linear estimation problem of signals has been one of the aspects approached in this kind of systems. We must first refer to Nahi [8], who, assuming a full knowledge of the state-space model and independence between the Bernoulli random variables, proposed a Riccati-type recursive algorithm to solve this problem. Subsequently, Hadidi and Schwartz [4] generalized this study considering that the variables modelling the uncertainty are not necessarily independent; they proved that the recursive property of the estimators is not always guaranteed and they established a necessary and sufficient condition for it. In other sense, assuming that the state model of the signal is not known but only using a factorization of the signal autocovariance function in a semi-degenerate kernel form, recently, Nakamori et al. [10],[11],[12] have proposed recursive algorithms to solve the LMSE linear estimation problem for the case of independent Bernoulli random variables as well as assuming uncertainty not necessarily independent.

By using also covariance information, in this paper we present a Chandrasekhar and a Riccati-type algorithm as solution of the LMSE linear filtering problem of wide-sense stationary signals from uncertain observations perturbed by white and coloured additive noises with uncertainty not necessarily independent. The comparison between both algorithms shows the computational advantages of the Chandrasekhar-type algorithm over the Riccati-type one. These advantages are clarified in a numerical example on estimation of signals transmitted in multichannel.

2. The LMSE linear filtering problem

Let us consider a discrete-time scalar observation equation described by

$$y(k) = u(k)z(k) + v(k) + v_0(k), \qquad z(k) = Hx(k) \tag{1}$$

where $y(k)$ represents the observation of the signal, $z(k)$, perturbed by a multiplicative noise, $u(k)$, and by white and coloured additive noises, $v(k)$ and $v_0(k)$, respectively; the signal is expressed as a linear combination of the components of the n-dimensional state vector, $x(k)$.

We are interested in analyzing the LMSE linear filtering problem of the signal, $z(k)$; for this purpose, we will assume the following hypotheses on the processes involved in the observation equation (1):

H1. The signal process $\{z(k);\ k \geq 0\}$ is a zero-mean wide-sense stationary process whose autocovariance function, $K_z(k,s) = E[z(k)z(s)] = K_z(k-s)$, can be factorized as $K_z(k,s) = H\Phi K_{xz}(k-1,s)$, where Φ denotes the system matrix in the state-space model of $x(k)$ and $K_{xz}(k-1,s)$ represents the crosscovariance function of the state $x(k-1)$ and the signal $z(s)$.

H2. The additive noise $\{v(k);\ k \geq 0\}$ is a zero-mean wide-sense stationary white process with autocovariance function $E[v(k)v(s)] = R\delta_K(k-s)$, being δ_K the Kronecker delta function.

H3. The coloured additive noise $\{v_0(k);\ k \geq 0\}$ is also a zero-mean wide-sense stationary process, with autocovariance function $K_{v_0}(k,s) = E[v_0(k)v_0(s)] = K_{v_0}(k-s)$, which can be factorized as $K_{v_0}(k,s) = \Phi_0 K_{v_0}(k-1,s)$, being Φ_0 the system matrix in the state-space model of $v_0(k)$.

H4. The multiplicative noise $\{u(k);\ k \geq 0\}$ describing the uncertainty in the observations is a sequence of identically distributed Bernoulli random variables with initial probability vector $(1-p,p)^T$ and conditional probability matrix $P(k/j)$. The $(2,2)$-element of this matrix is assumed to be independent of k and j, that is,

$$p_{22}(k/j) = P(u(k)=1/u(j)=1) = p_{22}, \quad k \neq j.$$

Consequently, the necessary and sufficient condition for the existence of recursive linear estimators proposed by Hadidi and Schwartz [4] (that is, $p_{22}(k/j)$ is independent of j for all $j<k$) is satisfied. Moreover, under these considerations, it is clear that

$$E\left[u(k)u(j)\right] = \begin{cases} p, & \text{if } k=j \\ p\,p_{22}, & \text{if } k\neq j \end{cases}$$

H5. The state and the noise processes, i.e. $\{x(k);\ k\geq 0\}$, $\{u(k);\ k\geq 0\}$, $\{v(k);\ k\geq 0\}$ and $\{v_0(k);\ k\geq 0\}$, are mutually independent.

From these hypotheses, we have approached the LMSE linear estimation problem of the signal $z(k)$ given the observations until time k, $\{y(1),\dots,y(k)\}$; as a result, we have obtained two recursive algorithms to calculate this estimator, denoted by $\widehat{z}(k,k)$. Next, we will specify how the algorithms have been derived and we will also establish some comparisons between them.

First of all, it can be easily observed from (1), that the filter of the signal is given by $\widehat{z}(k,k) = H\widehat{x}(k,k)$, where $\widehat{x}(k,k)$ represents the state filter. For this reason, our interest is focused on obtaining an algorithm for $\widehat{x}(k,k)$, which can be expressed as

$$\widehat{x}(k,k) = \sum_{i=1}^{k} h(k,i)y(i)$$

where $h(k,i),\ i=1,\dots,k$ denotes the impulse-response function.

As a consequence of the Orthogonal Projection Lemma (OPL) (see e.g. [5]), $\widehat{x}(k,k)$ satisfies the Wiener-Hopf equation which, from the hypotheses on the model, is given by

$$\begin{aligned} &h(k,s)W = pK_{xz}(k,s) - \sum_{i=1}^{k} h(k,i)\overline{K}(i,s), \qquad 1\leq s\leq k \\ &W = R + p(1-p_{22})HK_{xz}(0), \quad \overline{K}(i,s) = pp_{22}HK_{xz}(i,s) + K_{v_0}(i,s). \end{aligned} \tag{2}$$

Moreover, the OPL also guarantees that $\widehat{x}(k,k)$ can be expressed as

$$\widehat{x}(k,k) = \widehat{x}(k,k-1) + h(k,k)[y(k) - \widehat{y}(k,k-1)]$$

where $\widehat{x}(k,k-1)$ and $\widehat{y}(k,k-1)$ represent the LMSE one-stage linear predictors of $x(k)$ and $y(k)$, respectively. Taking again into account the OPL and the hypotheses on the model, it is deduced that

$$\begin{aligned}\widehat{x}(k,k-1) &= \Phi\widehat{x}(k-1,k-1)\\ \widehat{y}(k,k-1) &= p_{22}H\Phi\widehat{x}(k-1,k-1) + \Phi_0\widehat{v}_0(k-1,k-1)\end{aligned}$$

where $\widehat{v}_0(k-1,k-1)$ represents the LMSE linear filter of $v_0(k-1)$. Similarly to the above reasoning, denoting now by $g(k,i)$, $i = 1,\dots,k$ the impulse-response function, the Wiener-Hopf equation associated with $\widehat{v}_0(k,k)$ is given by

$$g(k,s)W = K_{v_0}(k,s) - \sum_{i=1}^{k} g(k,i)\overline{K}(i,s), \ 1 \le s \le k \tag{3}$$

and the filter of the coloured noise verifies

$$\widehat{v}_0(k,k) = \Phi_0\widehat{v}_0(k-1,k-1) + g(k,k)[y(k) - \widehat{y}(k,k-1)].$$

As a result of these considerations, let us observe that the algorithm for $\widehat{x}(k,k)$ only requires the determination of formulas for the filtering gains, $h(k,k)$ and $g(k,k)$. Next, we show different manners to calculate them and we establish comparisons between the corresponding algorithms.

3. Filtering algorithms

In the following theorems, we present two algorithms to calculate the LMSE linear filter of the signal, $\widehat{z}(k,k)$; the difference between them is that the filtering gains are calculated by a different way: in one of them, they are obtained from Chandrasekhar-type equations whereas, in the other, Riccati-type equations are used.

THEOREM 1. *Given the observation equation (1) with the hypotheses H1-H5, the filter of the signal is given by $\widehat{z}(k,k) = H\widehat{x}(k,k)$, where $\widehat{x}(k,k)$ is recursively obtained from the following relations*

$$\begin{aligned}\widehat{x}(k,k) &= \Phi\,\widehat{x}(k-1,k-1)\\ &\quad + h(k,k)\,[y(k) - p_{22}H\Phi\,\widehat{x}(k-1,k-1) - \Phi_0\,\widehat{v}_0(k-1,k-1)]\,;\\ \widehat{x}(1,1) &= h(1,1)y(1)\end{aligned} \tag{4}$$

$$\begin{aligned} \widehat{v}_0(k,k) &= \Phi_0 \widehat{v}_0(k-1,k-1) \\ &\quad + g(k,k)\left[y(k) - p_{22}H\Phi\,\widehat{x}(k-1,k-1) - \Phi_0\widehat{v}_0(k-1,k-1)\right]; \\ \widehat{v}_0(1,1) &= g(1,1)y(1). \end{aligned} \tag{5}$$

The filter gains, $h(k,k)$ and $g(k,k)$, are calculated as follows

$$\begin{aligned} h(k,k) = \Big[&h(k-1,k-1) - \Phi h(k-1,1)\left[p_{22}H\Phi h(k-1,1)\right. \\ &\left.+ \Phi_0 g(k-1,1)\right]\Big] \\ &\times \Big[1 - \left[p_{22}H\Phi h(k-1,1) + \Phi_0 g(k-1,1)\right]^2\Big]^{-1} \end{aligned} \tag{6}$$

$$\begin{aligned} g(k,k) = \Big[&g(k-1,k-1) - \Phi_0 g(k-1,1)\left[p_{22}H\Phi h(k-1,1)\right. \\ &\left.+\Phi_0 g(k-1,1)\right]\Big] \\ &\times \Big[1 - \left[p_{22}H\Phi h(k-1,1) + \Phi_0 g(k-1,1)\right]^2\Big]^{-1} \end{aligned} \tag{7}$$

where $h(k,1)$ and $g(k,1)$ satisfy the following relations

$$h(k,1) = \Phi h(k-1,1) - h(k,k)\left[p_{22}H\Phi h(k-1,1) + \Phi_0 g(k-1,1)\right] \tag{8}$$

$$g(k,1) = \Phi_0 g(k-1,1) - g(k,k)\left[p_{22}H\Phi h(k-1,1) + \Phi_0 g(k-1,1)\right] \tag{9}$$

with initial conditions

$$h(1,1) = pK_{xz}(0)\left[R + pHK_{xz}(0) + K_{v_0}(0)\right]^{-1} \tag{10}$$

$$g(1,1) = K_{v_0}(0)\left[R + pHK_{xz}(0) + K_{v_0}(0)\right]^{-1}. \tag{11}$$

Proof. Let us firstly observe that equations (4) and (5) have been already derived in the above section. Hence, we will next focus on deducing the recursive formulas for the filtering gains. So, if we replace k by $k-1$ and s by $s-1$ in (2) and subtract the resultant expression from (2), we obtain the following expression

$$\begin{aligned} \left[h(k,s) - h(k-1,s-1)\right]W &= -h(k,1)\overline{K}(1,s) \\ &\quad - \sum_{i=2}^{k}\left[h(k,i) - h(k-1,i-1)\right]\overline{K}(i,s), \quad 2 \le s \le k. \end{aligned}$$

Then, if we define a function J satisfying

$$J(k,s)W = \overline{K}(1,s) - \sum_{i=2}^{k} J(k,i)\overline{K}(i,s), \quad 2 \le s \le k \tag{12}$$

it is immediately obtained that

$$h(k,s) - h(k-1,s-1) = -h(k,1)J(k,s), \quad 2 \le s \le k. \tag{13}$$

Next, considering $k-1$, if we multiply (2) by $p_{22}H\Phi$ and (3) by Φ_0, and we add the resultant expressions, then we have, for $1 \le s \le k-1$,

$$\begin{aligned}[p_{22}H\Phi h(k-1,s) &+ \Phi_0 g(k-1,s)]\, W = \overline{K}(k,s) \\ &- \sum_{i=1}^{k-1} [p_{22}H\Phi h(k-1,i) + \Phi_0 g(k-1,i)]\, \overline{K}(i,s).\end{aligned} \tag{14}$$

Replacing s by $k-s+1$ in the above expression and taking into account that, from the stationary properties, $\overline{K}(i,s) = \overline{K}(s,i)$, it is immediately obtained that

$$\begin{aligned}[p_{22}H\Phi h(k-1,k-s+1) &+ \Phi_0 g(k-1,k-s+1)]\, W \\ &= \overline{K}(1,s) - \sum_{i=2}^{k} [p_{22}H\Phi h(k-1,k-i+1) \\ &\quad + \Phi_0 g(k-1,k-i+1)]\, \overline{K}(i,s), \quad 2 \le s \le k.\end{aligned}$$

Then, by comparing this equation with (12), it is clear that

$$J(k,s) = p_{22}H\Phi h(k-1,k-s+1) + \Phi_0 g(k-1,k-s+1), \quad 2 \le s \le k \tag{15}$$

and, hence, by putting $s = k$ in (15) and by substituting the resultant equation in (13) for $s = k$, we obtain

$$\begin{aligned}h(k,k) &- h(k-1,k-1) \\ &= -h(k,1)\,[p_{22}H\Phi h(k-1,1) + \Phi_0 g(k-1,1)]\,.\end{aligned} \tag{16}$$

Since this expression for $h(k,k)$ depends on $h(k,1)$, next we will derive a recursive formula for it. For this purpose, if we multiply (2) for $k-1$ by Φ and we subtract it from (2), we have

$$\begin{aligned}[h(k,s) &- \Phi h(k-1,s)]\, W = -h(k,k)\overline{K}(k,s) \\ &- \sum_{i=1}^{k-1} [h(k,i) - \Phi h(k-1,i)]\, \overline{K}(i,s), \;\; 1 \le s \le k-1.\end{aligned} \tag{17}$$

Next, if we compare (17) with (14), it is immediately deduced the following expression

$$\begin{aligned} &h(k,s) - \Phi h(k-1,s) \\ &\qquad = -h(k,k)\left[p_{22} H \Phi h(k-1,s) + \Phi_0 g(k-1,s)\right], \quad 1 \leq s \leq k-1 \end{aligned}$$

which leads to (8) by putting $s = 1$. Moreover, from (8) and (16), expression (6) is derived.

By following a similar reasoning, formulas (7) and (9) for $g(k,k)$ and $g(k,1)$ are obtained.

Finally, the initial conditions (10) and (11) are immediately derived by putting $k = s = 1$ in (2) and (3). ∎

Next we present the Riccati-type algorithm. Its proof is omitted since it can be easily obtained by following an analogous reasoning to that used by Nakamori et al. [12].

THEOREM 2. *If we consider the observation equation* (1) *satisfying the hypotheses H1-H5, the filter of the signal* $z(k)$ *is given by* $\widehat{z}(k,k) = H\widehat{x}(k,k)$, *where* $\widehat{x}(k,k)$ *can be calculated from the recursive relations*

$$\begin{aligned} \widehat{x}(k,k) &= \Phi\, \widehat{x}(k-1,k-1) \\ &\qquad + h(k,k)\left[y(k) - p_{22} H \Phi\, \widehat{x}(k-1,k-1) - \Phi_0\, \widehat{v}_0(k-1,k-1)\right]; \\ \widehat{x}(0,0) &= 0 \\ \widehat{v}_0(k,k) &= \Phi_0 \widehat{v}_0(k-1,k-1) \\ &\qquad + g(k,k)\left[y(k) - p_{22} H \Phi\, \widehat{x}(k-1,k-1) - \Phi_0\, \widehat{v}_0(k-1,k-1)\right]; \\ \widehat{v}_0(0,0) &= 0. \end{aligned}$$

The filter gains, $h(k,k)$ *and* $g(k,k)$, *are given by*

$$\begin{aligned} h(k,k) &= \left[p K_{xz}(k,k) - p_{22}\Phi S(k-1)\Phi^T H^T - \Phi_0 \Phi T(k-1)\right]\Pi^{-1}(k) \\ g(k,k) &= \left[K_{v_0}(k,k) - p_{22}\Phi_0 T^T(k-1)\Phi^T H^T - \Phi_0^2 U(k-1)\right]\Pi^{-1}(k) \end{aligned}$$

where $\Pi(k)$, $S(k)$, $T(k)$ *and* $U(k)$ *are calculated as follows*

$$\begin{aligned} \Pi(k) &= R + pK_{xz}^T(k,k)H^T + K_{v_0}(k,k) - p_{22}^2 H\Phi S(k-1)\Phi^T H^T \\ &\qquad - p_{22}\Phi_0 H \Phi T(k-1) - p_{22}\Phi_0 T^T(k-1)\Phi^T H^T - \Phi_0^2 U(k-1) \\ S(k) &= \Phi S(k-1)\Phi^T + h(k,k)\Pi(k)h^T(k,k);\ S(0) = 0 \\ T(k) &= \Phi_0 \Phi T(k-1) + g(k,k)h(k,k)\Pi(k);\ T(0) = 0 \\ U(k) &= \Phi_0^2 U(k-1) + g^2(k,k)\Pi(k);\ U(0) = 0. \end{aligned}$$

By comparing the algorithms presented in Theorems 1 and 2, it is observed that, since the state vector $x(k)$ has dimension n, the number of operations which have to be performed at each iteration of the Chandrasekhar-type algorithm is $3n+3$, whereas for the Riccati-type algorithm, are $3n+\frac{n(n+1)}{2}+4$ the operations needed. The reduction regarding the number of operations to perform, which is more significant as the dimension of the state vector is larger, implies that the computation time is smaller and, hence, that the Chandrasekhar-type algorithm is more advantageous than the Riccati-type one in a computational sense. These computational advantages can be clearly observed in the example that we present in the following section.

Finally, as a measure of the estimation accuracy, the filter error variance, $P(k,k) = E\left[\{z(k) - \widehat{z}(k,k)\}^2\right]$, can be calculated as

$$P(k,k) = H\left[K_{xz}(k,k) - S(k)H^T\right]$$

where $S(k)$ is given in Theorem 2.

4. Filtering of signals transmitted in multichannel

Let us consider the problem of estimating a wide-sense stationary scalar signal process, $\{z(k); k \geq 0\}$, which can be transmitted through several channels randomly picked. Specifically, in this example, we have considered four transmission channels (although the results can be easily generalized to an arbitrary number of channels); each of these channels is characterized by the following observation equation

$$\text{Channel } i: \; y(k) = u_i(k)z(k) + v(k) + v_0(k) \quad i = 1, \ldots, 4 \tag{18}$$

that is, in the i-th channel, the signal is perturbed by a multiplicative noise, $\{u_i(k); k \geq 0\}$, modelled by a sequence of independent Bernoulli random variables with $P(u_i(k) = 1) = p_i$ and by white and coloured additive noises, $\{v(k); k \geq 0\}$ and $\{v_0(k); k \geq 0\}$, respectively. We also assume that the signal is expressed as a function of the state vector $x(k)$ by means of the relation $z(k) = Hx(k)$.

Denoting by q_i the probability that the signal is transmitted by the i-th channel, the equation (18) can be rewritten as $y(k) = u(k)z(k) + v(k) + v_0(k)$, where $\{u(k); k \geq 0\}$ is a sequence of Bernoulli random variables with $p = P(u(k) = 1) = \sum_{i=1}^{4} p_i q_i$, for all $k \geq 0$, and conditional probability matrix given by

$$P(k/j) = \begin{pmatrix} \dfrac{\sum_{i=1}^4 q_i(1-p_i)^2}{\sum_{i=1}^4 q_i(1-p_i)} & \dfrac{\sum_{i=1}^4 q_i p_i(1-p_i)}{\sum_{i=1}^4 q_i(1-p_i)} \\ \dfrac{\sum_{i=1}^4 q_i p_i(1-p_i)}{\sum_{i=1}^4 q_i p_i} & \dfrac{\sum_{i=1}^4 q_i p_i^2}{\sum_{i=1}^4 q_i p_i} \end{pmatrix}, \ \forall k, j \geq 0.$$

Let us also suppose that the white additive noise, $\{v(k); k \geq 0\}$, is gaussian with zero mean and constant variance $R = 0.1^2$, and that the only available information on the signal is its autocovariance function defined by

$$\begin{aligned} K_z(i) = 0.25 &\left[\frac{(1-\alpha_2^2)(1-\alpha_3^2)(1-\alpha_2\alpha_3)\alpha_1^{i+2}}{(\alpha_2-\alpha_1)(\alpha_3-\alpha_1)} \right. \\ &+ \frac{(1-\alpha_1^2)(1-\alpha_3^2)(1-\alpha_1\alpha_3)\alpha_2^{i+2}}{(\alpha_1-\alpha_2)(\alpha_3-\alpha_2)} \\ &\left. + \frac{(1-\alpha_1^2)(1-\alpha_2^2)(1-\alpha_1\alpha_2)\alpha_3^{i+2}}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_3)} \right] \\ &\times \left[1 + \alpha_1\alpha_2 + \alpha_1\alpha_3 + \alpha_2\alpha_3 - \alpha_1\alpha_2\alpha_3(\alpha_1\alpha_2\alpha_3 + \alpha_1 + \alpha_2 + \alpha_3) \right]^{-1} \end{aligned}$$

being α_1^{-1}, α_2^{-1}, α_3^{-1}, the roots of the equation $1 + a_1 x + a_2 x^2 + a_3 x^3 = 0$, with $a_1 = -1.6$, $a_2 = 0.76$ and $a_3 = -0.096$. Then, by using the factorization technique proposed by Nakamori et al. [10], it is obtained

$$H = (1 \ \ 0 \ \ 0), \quad \Phi = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.096 & -0.76 & 1.6 \end{pmatrix}, \quad K_{xz}(0) = \begin{pmatrix} 0.25 \\ 0.239 \\ 0.2154 \end{pmatrix}.$$

In a similar way, if we consider that the autocovariance function of the coloured noise is given by

$$K_{v_0}(i) = 0.25(-0.2)^i, \quad i \geq 0$$

from the factorization technique we have that $\Phi_0 = -0.2$.

Let us note that, for applying the algorithms, it is necessary to know the values of q_i and p_i for $i = 1, 2, 3, 4$, which lead to the values of p and $p_{22} = P(u(k) = 1/u(k-1) = 1)$. In this example, we have considered $q_1 = 0.1$, $q_2 = 0.15$, $q_3 = 0.35$, $q_4 = 0.4$ and different values of p_i, which yield to the fixed value $p = 0.75$, and different values of p_{22}, specifically:

$\cdot p_1 = p_2 = p_3 = p_4 = 0.75 \quad \Rightarrow \quad p = p_{22} = 0.75$

$\cdot p_1 = p_2 = p_3 = 0.95; \ p_4 = 0.45 \quad \Rightarrow \quad p = 0.75; \ p_{22} = 0.83$

$\cdot p_1 = p_2 = 0.15; \ p_3 = p_4 = 0.95 \quad \Rightarrow \quad p = 0.75; \ p_{22} = 0.91$

$\cdot p_1 = p_2 = 0; \ p_3 = p_4 = 1 \quad \Rightarrow \quad p = 0.75; \ p_{22} = 1.$

We may remark that the algorithms corresponding to situation $p_{22} = 0.75$ coincide with that of case of uncertain observations with uncertainty modelled by independent random variables.

By considering the different values of p_{22} previously mentioned, we have performed 300 iterations of the Chandrasekhar-type algorithm and we have calculated the filtering error variances.

In Figures 1-4, the simulated signal and the filtering estimate are shown; the filtering error variances are drawn in Figure 5. These figures show that as p_{22} increases the estimates of the signal are worse.

Finally, we have calculated the ratio between the computation times of the Chandrasekhar-type algorithm and that corresponding to the Riccati-type algorithm; the results have been 0.4545, 0.2941, 0.5454 and 0.375 for $p_{22} = 0.75,\ 0.83,\ 0.91$ and 1, respectively. From these values, we conclude that the algorithm based on Chandrasekhar equations is computationally more efficient, since it reduces considerably the computation time.

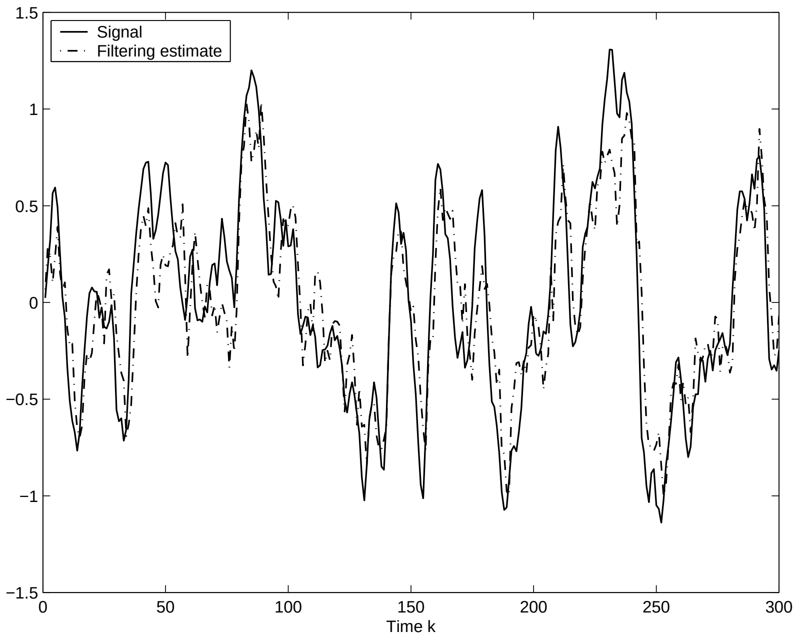

Figure 1: Signal and its filtering estimate for $p_{22} = 0.75$.

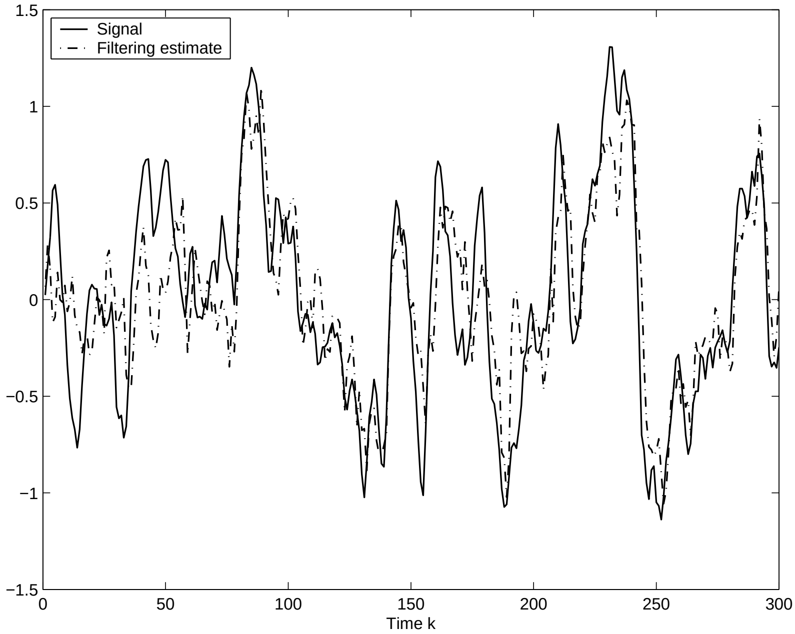

Figure 2: Signal and its filtering estimate for $p_{22} = 0.83$.

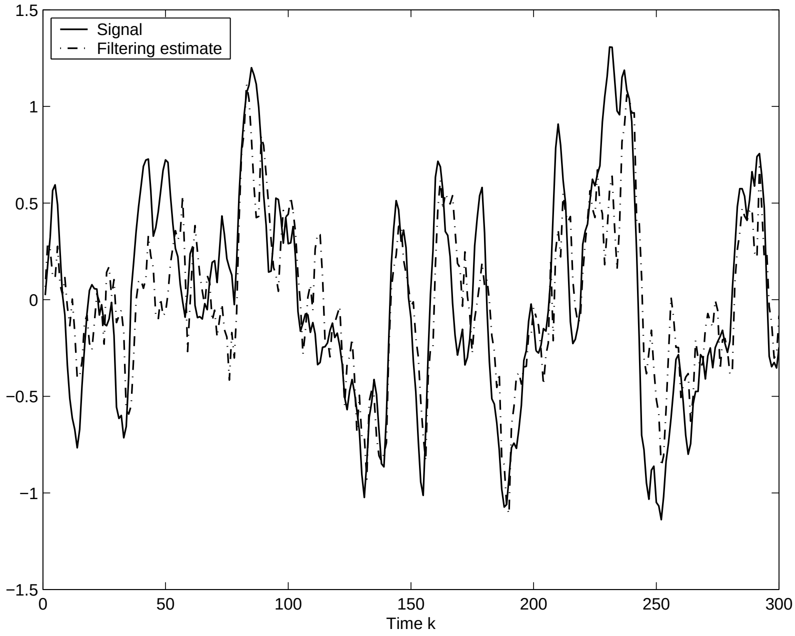

Figure 3: Signal and its filtering estimate for $p_{22} = 0.91$.

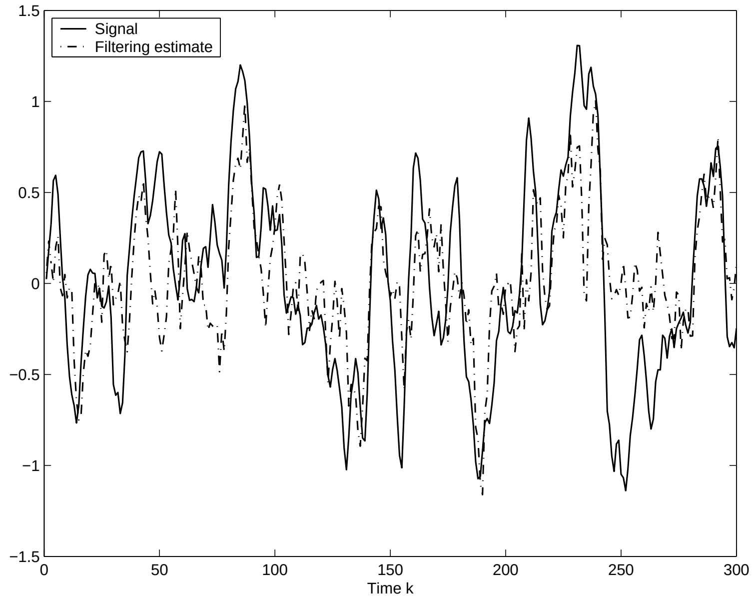

Figure 4: Signal and its filtering estimate for $p_{22} = 1$.

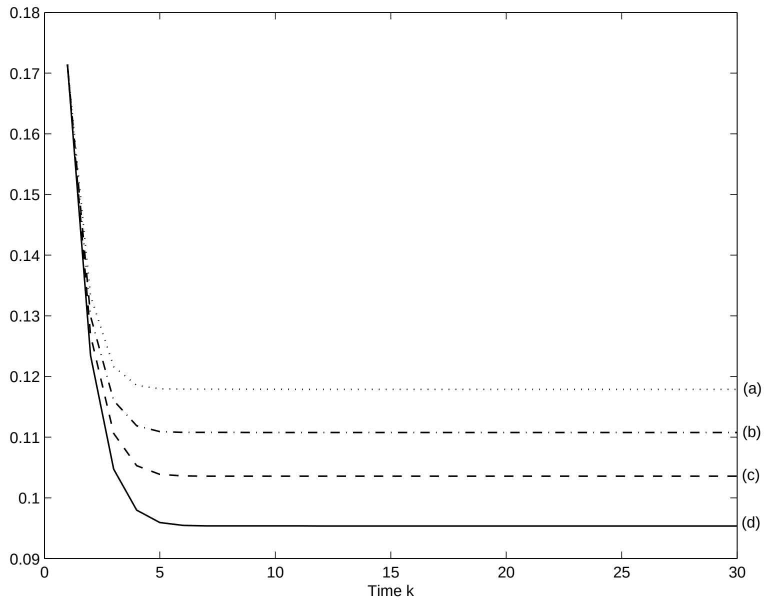

Figure 5: Filter error variances when (a) $p_{22} = 1$; (b) $p_{22} = 0.91$; (c) $p_{22} = 0.83$; (d) $p_{22} = 0.75$.

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