EQUIVARIANT UNFOLDINGS OF STRATIFIED PSEUDOMANIFOLDS

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Abstract

The Intersection Cohomology with differential forms defined on stratified pseudomanifolds, introduced by Brasselet [1], Hector and Saralegi [5]; uses the unfolding given by Verona [7] as an auxiliary tool. Broadly speaking, an unfolding over a stratified pseudomanifold X is a smooth manifold X together with a continuous surjective map $\mathcal{L}_X : \widetilde{X} \to X$ such that the restriction to the regular part $\mathcal{L}_X : \mathcal{L}^{-1}(X - \Sigma) \to X - \Sigma$ is a finite trivial covering, and for each singular stratum S the restriction $\mathcal{L}_X : \mathcal{L}^{-1}(S) \to S$ is a smooth fiber bundle with fiber $L_S \times \mathbb{R}$ where L_S is the unfolding of the link L_S of the stratum S. The recursive method employed here turns this construction more difficult when the length of X is > 1. In this work we introduce a class \mathfrak{G} of G-transverse Thom-Mather stratified pseudomanifolds. An object of \mathfrak{G} is an unfoldable stratified pseudomanifold X with arbitrary length, endowed with the action of a compact Lie group G such that the stratification of Xis a refinement of the partition by orbit types (namely, the isotropy groups are constant over each stratum) and where the local conical structure is given by a slice of the action. Besides, we ask the stratification of X to be Thom-Mather compatible with the action, so there are equivariant tubular neighborhoods over the singular strata and these tubes are locally transverse to the action.

A fundamental property is that for any object $X \in \mathfrak{G}$ and $K \subset G$ a closed subgroup, the quotient space X/K is a G/K-transverse Thom-Mather stratified pseudomanifold, the quotient stratification being induced by the natural projection map. Since each smooth G-manifold is an object in \mathfrak{G} , this fact provides a rich source of examples of unfoldable stratified pseudomanifolds wich can be obtained starting on a smooth manifold.

References

- BRASSELET, J.P.; Hector, G. & Saralegi, M. Théorème de de Rham pour les varietés stratifiées. Ann. Global Anal. Geom 9 (1991), pp. 201-243.
- [2] GORESKY, M. & MACPHERSON, R. Intersection Homology Theory. Topology 19, 135-162 (1980).
- [3] GORESKY, M. & MACPHERSON, R. Intersection Homology II. Invent. Math. 71, 77-129 (1983).
- [4] HECTOR, G. & SARALEGI, M. Intersection Cohomology of S³-actions. Trans. Amer. Math. Soc. 338, 263-288 (1993).
- [5] SARALEGI, M. Homological Properties of Stratified Spaces. Illinois J. Math. 38, 47-70 (1994).
- [6] VERONA, A. Le théoreme de DeRham pour les préstratifications abstraites. C.R. Acad. Sci. Paris 273 (1971), 886-889.
- [7] VERONA, A. Stratified mappings, structure and triangulability. Lecture Notes in Math. Springer-Verlag. Vol. 1102. Springer-Verlag. New York-Heidelberg- Berlin (1984).

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