## RESEARCH ARTICLE

# QCircularStats: A QGIS-Plugin for Evaluation Bidimensional Data by Circular Statistics 

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#### Abstract

Circular statistics have been developed and used in a wide range of branches of knowledge related to the study and observation of the Earth, such as physical, geographical, ecological, and biological sciences. Both to facilitate its use by a less specialised community and to extend its analysis within the research community, this work presents the application of circular statistics in QGIS, an open-source Geographic Information Systems, through the design and implementation of a novel and complete plugin, named QCircularStats. This free and open-source plugin provides a comprehensive, understandable, and visual way to conduct circular analysis over remote sensing data. The behaviour of the proposed QCircularStats is evaluated over real satellite data. Particularly, data on wind speed and direction over the oceans collected by the microwave sensor allocated on the QuikSCAT satellite have been considered. Results obtained demonstrate the performance, functionality, and versatility of the implemented plugin.


INDEX TERMS Circular statistics, circular data, python, QGIS pluging.

## I. INTRODUCTION

Since their inception based on the digitisation of the geographic space conceptualisations and physical maps, Geographic Information Systems (GIS) have been an important instrument for a wide range of socio-economic, industrial and scientific activities, providing beneficial support for the analysis of a wide variety of phenomena with a spatial (geographic) component. In fact, those disciplines related to the study, understanding and exploitation of the Earth, whether they are physical, geographical, ecological and biological sciences, or industrial processes and economic activities such as mining, agronomy or urban planning, have had to address the representation, processing and manipulation of data with a spatial dependence that defines a very specific spatio-temporal framework. In this context, GIS have been used in input, output, storage, manipulation,

[^0]treatment, and analysis of remote sensing spatial data. Due to their excellent results as analytical tools, GIS are being increasingly used in applications for natural and environmental resource settings, transportation, exchange, tourism, and trade, among others. Furthermore, the use of GIS is growing along with major developments in computer and telecommunications technologies, offering the possibility of integrating new geo-positioning data (GNSS) and novel remote sensing technologies.

GIS are popular tools for the representation and analysis of two-dimensional spatial data. In this context, the literature offers a vast amount of works that take advantage of GIS features to perform remotely sensed data analysis, such as the evaluation of landslides [1]; the mapping, assessment, and quantification of the societal values of ecosystem services [2]; aquatic ecosystem models [3]; environmental indices computing using Landsat data [4]; hydraulic simulation tools [5]; soil and water assessment [6]; modelling of rockfall run-out [7]; evapo-transpiration predictions [8]; positional

TABLE 1. Some recent QGIS software plugins.

| Acronym | Name | Year |
| :---: | :---: | :---: |
| FSLAM | Fast Shallow Landslide Assessment Model | 2022 |
| SolVES | Social Values for Ecosystem Services | 2022 |
| QWET | QGIS Water Ecosystems Tool | 2021 |
| Q-LIP | Qgis-Landsat Indices plugin | 2021 |
| QPROTO | Modeling of rockfall runout | 2021 |
| QWaterModel | Evapotranspiration predictions | 2020 |
| EPANET | Evaluating water distribution systems performance | 2019 |
| SWAT | Soil and Water Assessment Tool | 2018 |

accuracy analysis of satellite imagery [9], among many others. In particular, the Quantum GIS (QGIS) is one of the most popular and widely used GIS due to its multi-purpose nature, its visualisation characteristics and its flexibility and versatility in managing, editing, and analysing data, as well as in the designing of maps and graphs. As a matter of fact, this software was developed as an open-source system, making it possible to develop, implement and incorporate specific plugins that extend its processing capabilities. Part of its great success is due to the possibility of developing, implementing, and incorporating specific plugins that extend the processing capabilities of the GIS. For instance, Table 1 shows some recently developed and published plugins, where data is analysed for different purposes.

Statistics is a vast field of study. It comprises both descriptive and inferential methods. While descriptive statistics aim to present and describe data, inferential statistics are concerned with its behaviour and probability. However, when working with the latter, statistics may be considered a nonexact science. Furthermore, statistics can be classified based on the number of dimensions it analyses, such as onedimensional (1D) or directional statistics. Directional statistics, which involves analysing data in circular, two-dimension (2D) or spherical, three-dimension (3D) space, plays a core role in fields where the analysis of directions is relevant. As such, this work focuses on directional statistics, which is traditionally considered a more advanced level of statistical analysis than 1D statistics.

In this context, different types of data can be used to analyse and develop environmental studies, out of which the most common are directional data. These are characterised by their definition in a circle or globe, i.e. data pertains to angular measurements (directions) composed of and angle or an orientation [10]. Indeed, these angular measurements are understood as points on the boundary of a circle (i.e., the circumference) or on the surface of a sphere, depending on the domain. This has several implications that make the processing and extraction of useful information for the end user very specific. This field of statistics has become particularly relevant over time and as pointed before, it can be divided into circular and spherical data. This work focuses on the former, circular data, which is defined in 2D space, and is expressed in angles/directions on a circle [10], [11], thus referring to orientation in a plane. Moreover, circular data generally follows a Von Mises distribution [12]; [13] and is widely used in data analysis.


FIGURE 1. Graphical representation of circular data provided by the proposed QCircularStats plugin. In particular, real wind data is depicted, representing (a) the angles on the unit circle and (b) the corresponding modules (wind force) of each observed sample.

## A. DRAWBACKS IN THE USE OF DIRECTIONAL STATISTICS

The use of directional data can be found in multiple branches of knowledge regarding earth sciences [14], as varied as geology [15], [16], geomatics [17], biology [18], [19], [20], oceanographic [21], or meteorology areas [22], [23]. Moreover, directional data has proven to be highly useful in a broad range of applications with a focus on the analysis of cyclical patterns, such as the study of migratory movements [24], the measurement of winds [25], [26], and the study of grooves and striations direction on fault planes, among many others. Furthermore, this kind of two-dimensional data can be found to convey and store the information of certain phenomena related to other fields, such as image processing [27], [28], physics [29], medicine [30], [31], [32], neurology [33], cognitive and experimental psychology studies [34], [35] or in machine learning data processing methods [36], [37].

Circular statistics gathers a number of methods and techniques designed for the processing and management of circular data, these $\hat{A}$ consisting of angles, axes, vectors and rotations. Key concepts in circular statistics include the circular mean, circular variance, and circular covariance, which are used to describe the central tendency, spread, and relationships between circular data, for instance. These methods are included on libraries, software packages and tools to exploit the rich information contained into circular data. Nevertheless, most of them derive from specialised fields of study, and neither they nor proprietary software integrate information in a way that is suitable for two-dimensional data analysis. For instance, [38] presented a circular toolbox called CircStat, providing analytical methods for the treatment of directional data. Notwithstanding its valuable contribution, the toolbox is based on a non-free software environment, MATLAB, which limits its public use. Other examples are [39] and [40], where a MATLAB script for the calculation of descriptive statistics, inference on azimuthal parameters and the circular variables correlation is presented. Nevertheless, MATLAB still has some problems in addition to the licence of use, and does not include some important functions, like algorithms to compute the function of a Von Mises distribution [10].

Additionally, there are some statistical software packages, which have their own extension for directional statistics
analysis, such as Stata and its open source CircStats package developed by [41] and [42]. Nevertheless, and although it is not sold in modules, allowing access to all functionalities in a single package, the Stata licence is not free, which poses an economic barrier for end-users.

By the same token, the R environment, a public free software designed to perform statistical computation and produce descriptive graphics that features several circular statistics tools, is used by a significant number of directional statistics packages. For instance, [43] proposed the bpnreg, a Bayesian model to perform analysis on longitudinal circular data based on the projected normal distribution. Also, [44] and [45] developed and improved the circular package for R , inspired by the previous research conducted by [46]. Previous studies conducted at University of Extremadura resulted in two R-based packages: the VecStatGraphs $2 D$ tool for circular statistics presented in [17], and the VecStatGraphs3D for spherical statistics implemented in [47]. Unfortunately, R suffers from a lack of flexibility and adaptability by focusing wholly on the area of mathematical analysis. A major drawback of these packages is also that there are few programs that offer a R language Application Programming Interface (API).

Along with R-based plugins, a number of Python-based libraries have been created that may be used to handle directional data. Examples of these libraries are SciPy, NumPy, and SciKit-Learn. Much as the Python language is particularly easy to use and quick to learn, these Python-based libraries, packages and tools, however, are highly specialised in a limited number of domains, and their use is $\hat{A}$ scarce. Consequently, they seem unsuitable for generic analyses that employ circular data.

To assist the calculation of circular statistics for nonspecialist users, this work proposed a new open-source tool, called QCircularStats. This is a novel open-source plugin integrated in QGIS for the statistical management, study and analysis of 2-D direction vectors. This new plugin is built in Python as a flexible and easy-to-use solution, incorporating circular statistics operations, additional graphical analysis, and techniques to offer a more versatile usage. Furthermore, an intuitive and user-friendly graphic user interface (GUI) has been created to make it easier for users to understand how this plugin operates and illustrate its possibilities within the statistics sector.

In this regard, the main objectives of this work can be stated as: (i) to present and describe in detail the proposed PyQGISbased plugin, which is inspired by and based on the code of the PyCircularStats library, called QCircularStats; (ii) to describe all results associated with the presented version, including 12 linear statistical parameters, 9 angular statistical parameters, 2 tests of uniformity, and mainly 7 circular statistical plots, and (iii) to apply the QCircularstats plugin to a target area in a widely used remote sensing dataset, with the aim of testing the behaviour of the plugin in a scientific context, by analysing the module and the orientation of the studied dataset.

To clearly present the proposed QCircularStats, the present work is organised as follows. Section II describes circular data and its related statistics with a proposed improvement of the two-dimensional directional non-unitlength vectors statistical analysis. Section III shows the technical features. Section IV presents an experiment conducted to test the utility of QCircularStats with the details of the dataset used. Finally, Section V summarises the conducted work and describes the final conclusions.

## II. CIRCULAR DATA AND CIRCULAR STATISTICS

As stated above, two- and three-dimensional directional data is identified by its definition on a circular or spherical domain, respectively, to measure angles, directions, phases, or periodic events. In particular, this paper focuses on circular data, which is the simplest type of directional data [11]. It is defined as points located on the circumference of the unit circle, i.e., in contrast to linear data supported by the real line, the support of circular data is the unit circle. It is noteworthy that, based on a Cartesian system with perpendicular axes X and Y and origin at $O$, each point on the plane is expressed by the rectangular coordinates $(x, y)$. By the same token, the points can be defined in the polar system as the pair $(\theta, d)$, where the former defines the angle, which is expressed in radians measured in the range 0 to $2 \pi$, or in degrees measured in the range $0^{\circ}$ to $360^{\circ}$, and the latter is the distance between the origin of the system O and the target point. In this sense, the azimuth indicates the direction of rotation of the angle taking the horizontal plane as reference. In this sense, polar representation is normally used to represent circular data, where $d$ is 1 in the unit circle or is representing some measured quantity. For instance, in Fig. 1(a) circular observations regarding wind data are represented by their angles, whilst in 1 (b), the wind force is represented graphically according to the modulus of the observations. With $d=1$, the conversion to rectangular coordinates is straightforward using trigonometric identities, i.e., $x=\cos \theta$ and $y=\sin \theta$.

As the fundamental property of circular data is that they are taken around a circle, there is no distinction between angles $\theta$ and $\theta+2 \pi$ (radians). Consequently, circular data has periodic nature with no true zero, that is, data is cyclic, without a beginning or end along the circle. Similar to Gaussian distribution for linear data, circular data normal distribution could follow the Von Mises distribution, which is the most usually adopted symmetric unimodal probability distribution for describing and modelling circular data, as it is mathematically simpler and easier to use. Fig. 2 depicts the traditional visualisation of the Von Mises function in cartesian and polar coordinates. Its probability density function (pdf) for the random angular variable $\theta, p(\theta ; \mu, k)$ is expressed by Eq. (1):

$$
\begin{align*}
p(\theta ; \mu, k) & =\frac{1}{2 \pi I_{0}(k)} \cdot e^{k \cos (\theta-\mu)}, \\
\text { with } 0 & \leq \theta \leq 2 \pi, \quad \mu<2 \pi, 0 \leq k<\infty \tag{1}
\end{align*}
$$



FIGURE 2. Graphical representation of the continuous probability Von Mises distribution on the circle, following Cartesian (a) and polar (b) coordinates. The support ranges from $-180^{\circ}$ to $\mathbf{1 8 0}^{\circ}$ with zero-mean $\mu=0$, whilst different values for concentration parameter $\boldsymbol{k}$, i.e., $\mathbf{0}$ (blue line), 1 (yellow line), $\mathbf{2}$ (green line), and $\mathbf{3}$ (red line) have been taken into consideration. It is worth noting that the value of $k$ determines the uniformity of the function, thus, the smaller it is, the closer the distribution is to the uniform-shape, whilst as $k$ increases, a normal distribution is approached in the specified angle with mean $\mu$ and variance $1 / k$.

TABLE 2. Parameters of linear statistic for circular analysis.

| Name | Equation |
| :---: | :---: |
| Number and set of samples | $N,\left\{d_{1}, \ldots, d_{N}\right\}$ |
| Maximum value | $\alpha_{\text {max }}=\max \left(d_{1}, \ldots, d_{N}\right)$ |
| Minimum value | $\alpha_{\text {min }}=\min \left(d_{1}, \ldots, d_{N}\right)$ |
| Range | $\alpha_{\text {max }}-\alpha_{\text {min }}$ |
| Arithmetic mean (a.k.a. population mean) | $\bar{X}=\frac{1}{N} \sum_{i=1}^{N} d_{i}$ |
| Unbiased Sample Variance and Population Variance | $S_{N}^{2}=\frac{1}{N-1} \sum_{i=1}^{N}\left(d_{i}-\bar{X}\right)^{2}, \sigma^{2}=\frac{1}{N} \sum_{i=1}^{N}\left(d_{i}-\bar{X}\right)^{2}$ |
| Sample and Population Standard Deviation | $S=\sqrt{\frac{1}{N-1} \sum_{i=1}^{N}\left(d_{i}-\bar{X}\right)^{2}}, \sigma=\sqrt{\frac{1}{N} \sum_{i=1}^{N}\left(d_{i}-\bar{X}\right)^{2}}$ |
| Standard Error | $S E_{\bar{X}}=\frac{\sigma}{\sqrt{N}}$ |
| Skewness coefficient | $g_{1}=\frac{m_{3}}{m_{2}^{3 / 2}} \text { where: } m_{n}=\frac{1}{N} \sum_{i=1}^{N}\left(d_{i}-\bar{X}\right)^{n}$ |
| Kurtosis coefficient | $g_{2}=\frac{m_{4}}{m_{2}^{2}} \text { where: } m_{n}=\frac{1}{N} \sum_{i=1}^{N}\left(d_{i}-\bar{X}\right)^{n}$ |

where parameters $\mu$ and $k$ are analogous to the mean and variance values in the normal distribution. Indeed $\mu$ is the mean direction, indicating the point where the pdf reaches its maximum value, whilst $k$ is the positive concentration parameter that quantifies the data dispersion, i.e., $1 / k$ could be roughly understood as the circular equivalent of the variance parameter ${ }^{1}$, and depends on the uniformity of the distribution.

[^1]Therefore, a zero-value of $k$ indicates that $p(\theta)$ is constant, i.e., when $k=0$ then $p(\theta)=1 / 2 \pi$ for all angles on the circle as the density is constant in any sector of the circle, bounded by the arc of the circumference together with its two corresponding radii, and the distribution is uniform. On the contrary, high concentrations imply low data variance, i.e., when $k>0$, the $p(\theta)$ has a maximum value in $\mu$ and a minimum value in $\mu+\pi$. As a result, the distribution becomes a normal distribution for large values of $k$. Moreover, the normalising factor $I_{0}(k)$ is the zeroth order Bessel function. In this regard, it is interesting to note that the modified Bessel

TABLE 3. Parameters of circular statistic for circular analysis.

| Name | Equation |
| :---: | :---: |
| Circular Mean Direction (mean azimuth) | $\bar{\theta}=\arctan 2(\bar{S}, \bar{C})$, with $\bar{S}=\frac{1}{N} \sum_{i=1}^{N} \sin \theta_{i}, \bar{C}=\frac{1}{N} \sum_{i=1}^{N} \cos \theta_{i}$ |
| Mean length (mean module) | $\bar{R}=\sqrt{S^{2}+\bar{C}^{2}}$ s.t. $0 \leq \bar{R} \leq 1$, with $\bar{S}=\frac{1}{N} \sum_{i=1}^{N} \sin \theta_{i}, \bar{C}=\frac{1}{N} \sum_{i=1}^{N} \cos \theta_{i}$ |
| Circular variance | $V=1-\bar{R}$ s.t. $0 \leq V \leq 1$ |
| Circular standard deviation | $b=\frac{1}{N} \sum_{i=1}^{N} \sin 2(1-V)=\sqrt{-2 \ln (\bar{R})}$ |
| Skewness | $k=\frac{1}{N} \sum_{i=1}^{N} \cos 2\left(\phi_{i}\right)$ |
| Kurtosis | $p(\theta ; \mu, k)=\frac{1}{2 \pi I_{0}(k)} e^{k \cos (\theta-\mu)}$ |
| Von Mises parameter |  |

function of the first kind and order $n$ for any input $x$, could be defined as:

$$
\begin{equation*}
I_{n}(x)=\sum_{r=0}^{\inf } \frac{1}{(r+n)!r!}\left(\frac{x}{2}\right)^{2 r+n}, \quad n=0,1,2, \ldots \tag{2}
\end{equation*}
$$

Thus, when $n=0$,

$$
\begin{align*}
I_{0}(k) & =\sum_{r=0}^{\inf } \frac{1}{(r!)^{2}}\left(\frac{k}{2}\right)^{2 r} \\
& =\frac{1}{2 \pi} \int_{0}^{2 \pi} e^{k \cos \mu} d \mu \tag{3}
\end{align*}
$$

In this context, both linear and circular statistics measurements presented in Tables 2 and 3, respectively, are used to provide valuable information from circular data. Focusing on the former, in order to investigate the module/magnitude of the circular samples, linear statistics are employed. They provide descriptive statistics, such as the number of elements, the arithmetic average, the maximum and minimum value, various standard measures, and certain coefficients. Circular statistics, on the other part, are used to analyse the angular measurements, i.e., azimuths, and offer certain circular metrics, like the circular mean direction and circular standard deviation, among others. For instance, considering a set of $N$ circular observations, i.e., $\left\{\theta_{i}, d_{i}\right\}(i=1, \ldots, N)$, with angles $\theta_{1}, \ldots, \theta_{N}$ and corresponding modules $d_{1}, \ldots, d_{N}$, several linear and circular measures of centralisation, dispersion and shape can be extracted.

## A. CENTRALISATION MEASURES

Focusing on centralisation parameters, the average of the modules (the central tendency) can be obtained through the arithmetic mean as $\bar{X}=\frac{1}{N} \sum_{i=1}^{N} d_{i}$.

Furthermore, considering the rectangular coordinates of the circular observations $\left(\cos \theta_{i}, \sin \theta_{i}\right), \forall i \in[1, N]$, the mean circular direction $\bar{\theta}$ of a given set of circular samples
is easily obtained as $\bar{\theta}=\arctan 2(\bar{S}, \bar{C})$. More specifically:

$$
\bar{\theta}= \begin{cases}\tan ^{-1}(\bar{S} / \bar{C}) & \text { if } \bar{C} \geq 0  \tag{4}\\ \tan ^{-1}(\bar{S} / \bar{C})+\pi & \text { if } \bar{C}<0\end{cases}
$$

where $\bar{C}=C / N$ and $\bar{S}=S / N$ are the mean of cosines and sines, respectively. Accordingly, the total sum of cosines and sines are $C=\sum_{i} \cos \theta_{i}$ and $S=\sum_{i} \sin \theta_{i}$. Indeed ( $\bar{C}, \bar{S}$ ) denotes the Cartesian coordinates of the centre of mass, thus, trigonometric identities can be applied to express the conversion from Cartesian to polar coordinates, i.e.,

$$
\begin{align*}
\bar{C} & =\frac{1}{N} \sum_{i=1}^{N} \cos \theta_{i}=\bar{R} \cos \bar{\theta} \\
\bar{S} & =\frac{1}{N} \sum_{i=1}^{N} \sin \theta_{i}=\bar{R} \sin \bar{\theta} \tag{5}
\end{align*}
$$

From Eq. (5), the mean resultant length $\bar{R}$ (which varies between 0 and 1 when all samples $x_{i}$ are unit vectors, i.e., $\left.d_{i}=1, \forall i \in[1, N]\right)$ is obtained as: $\bar{R}=\sqrt{\bar{C}^{2}+\bar{S}^{2}}$.

## B. DISPERSION MEASURES

It is noteworthy that $\bar{R}$ varies directly with the dispersion of the data, i.e., the higher the $\bar{R}$, the greater the concentration of the data with smaller variations between each $\theta_{i}$. Thus, measures of circular dispersion, such as circular variance $V=1-\bar{R}$ (which is between 0 and 1 when all $x_{i}$ are unit vectors, too), can be obtained. In addition, some linear standard deviation measures are also provided to analyse the spread of the data distribution, such as the sample standard deviation (SSD) that depends upon the sample, considering the sample size $N-1$ and the sample mean $\bar{X}$, where a high sample variance indicates samples are scattered and far away from the mean, and the population standard deviation (PSD) that is calculated from every individual in the population.

## C. SHAPE MEASURES

Also, some shape measurements are provided, such as the Skewness coefficient for analysing the asymmetry of the


FIGURE 3. Flowchart of the proposed QCircularStats plugin.
distribution, i.e., the lack of symmetry understanding a symmetric distribution if its mean, median and mode coincide. In this regard, whilst variance provides information about the amount of variability, the Skewness coefficient gives the direction of variability, where a close to zero value of shows symmetry around the mean direction. Skewness coefficient can be obtained based on absolute measures by comparing mean against median and mode, or on relative measures by considering the second and third central moments ( $m_{3}$ and $m_{2}$ ). Particularly, the Karl Pearson Gamma coefficient $g_{1}$ is obtained to peoprly provide the direction of Skewness.

Moreover, the Kurtosis coefficient is considered to study the convexity of the distribution, i.e., whether the data is heavy-tailed or light-tailed with respect to the distribution i.e., a high Kurtosis value indicate that the data distribution has heavy tails, or outliers, whilst low Kurtosis values indicates a lack of outliers. Similar to Skewness, Kurtosis coefficient is based on central momentum of order $n$, which is defined as the mean of the deviations to the $n$-th power:

$$
\begin{equation*}
m_{n}=\frac{1}{N} \sum_{1}^{N}\left(d_{i}-\bar{X}\right)^{n} \tag{6}
\end{equation*}
$$

, i.e., the $n$-th moment is the mean difference between each sample and the distribution mean. Regarding the circular Skewness and Kurtosis coefficients, they are obtained taking into account the angle between the $i$-th sample and the median distribution angle, denoted as $\phi_{i}$.

Von Mises concentration parameter $k$ is provided too, the value of which is used to analyse the distribution of circular data in an analogous way to the normal distribution in linear analysis, i.e., $k=0$ corresponds to the uniform distribution, and the larger $k$, the more concentrated the distribution is around its mean.

In addition, the periodicity Rayleigh test [11] conducts the uniformity analysis to determine the randomness of circular irregularly sampled data, considering Eq. (7):

$$
\begin{equation*}
z=N \bar{R}^{2}=\frac{1}{N}\left[\left(\sum_{i=1}^{N} \cos \theta_{i}\right)^{2}+\left(\sum_{i=1}^{N} \sin \theta_{i}\right)^{2}\right] \tag{7}
\end{equation*}
$$

where $N$ is the size of the problem (i.e., the number of vectors) and $\bar{R}$ is the mean resultant vector of $N$ samples [48]. Also, Rao Score test ${ }^{2}$ [19] is used for assessing the homogeneity and dispersion of circular datasets, considering that, given a uniform distribution, the spacing between samples should be $360^{\circ} / N$.

Tables 2 and 3 summarise some of the most widely used linear and circular statistic parameters, to analyze the modules, and azimuths of circular samples. More detailed information on these measurements can be consulted at chapter 2 and 4 of [11] and chapter 2 of [50].

## III. TECHNICAL IMPLEMENTATION

The proposed QCircularStats is implemented as a free and open-source software, and takes over the philosophy of the aforementioned plugins, seeking to empower generic users and bring complex circular statistics calculations to completion. The proposed plugin is Python-based and allows users to perform circular statistics calculations obtaining both numerical results along with graphical support; its functionalities are written in Python, an interpreted programming language very popular in the scientific community due to its flexibility and simplicity, whilst the graphical user interface (GUI) is developed and implemented through the "what-you-see-is-what-you-get" (WYSIWYG)-based Qt Designer,

[^2]

FIGURE 4. Interface of the proposed $Q C i r c u l a r S t a t s$ plugin.
due to its ease of use and speed of production. The developed QCircularStats is compatible with QGIS 3 version. Furthermore, all the functionalities of the proposed QCircularStats can be run in standard QGIS Desktop with no additional prerequisites nor requirements.

Fig. 3 shows an operational scheme of the developed QCircularStats plugin, whilst Fig. 4 depicts its graphical user interface. The two circular data entry possibilities are shown at the bottom of Fig. 4, which are explained in detail in section III-A. As it can be observed on Fig. 4, on the left side of the GUI screen of this plugin, the desired output must be individually selected for the result to be displayed on the screen. The output possibilities, both numerical and graphic, are also detailed in section III-B.

## A. INPUT DATA FORMAT

As mentioned before, the data input can be represented through two possibilities: i) through raster layers, where each layer represents a circular parameter, i.e., modules and azimuths and ii) through text format data.
In the pop-up screen of the interface, two possible ways to enter the data are provided. In Fig. 4, it can be observed these two possible ways in the implemented interface. On the one hand, at the bottom of this pop-up screen, the option 'Load Map' is provided, where two raster layers, a module layer and azimuth layer respectively, are selected. Such layers must be open in the QGIS project; the raster layer that represents the modules is read in 'Modules'; and similarly, the layer with the azimuth values is read in 'Azimuts' (see Fig. 4).

On the other hand, if raster files are not avaible, the data can be loaded through plain text files (.txt) through the 'Select file' option. In this case, the data entry has three different representations: a) cartesian; b) polar and c) incremental. In the case of plain text files, two-column and four-column formats with tabs separating the data should be used as input files: a

TABLE 4. Examples of input format files supported by $Q C i r c u l a r S t a t s$.

| Cartesian |  |  |  | Incremental |  | Polar |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $x_{o}$ | $y_{o}$ | $x_{d}$ | $y_{d}$ | $D_{x}$ | $D_{x}$ | $d$ | $\theta$ |
| 16.3724 | 37.3681 | 5.7534 | 5.5101 | 10.6190 | 31.8580 | 33.5812 | 18.4344 |
| 31.7555 | 0.6522 | 5.2075 | 5.9622 | 26.5480 | -5.3100 | 27.0738 | 101.3108 |
| 32.0306 | 47.7206 | 5.4826 | 5.2436 | 26.5480 | 42.4770 | 50.0908 | 32.0053 |
| 21.2150 | 0.2490 | 5.2860 | 5.5580 | 15.9290 | -5.3090 | 16.7904 | 108.4328 |
| 32.2584 | -37.1992 | 5.7104 | 5.2778 | 26.5480 | -42.4770 | 50.0908 | 147.9947 |
| 32.3682 | -21.0106 | 5.8192 | 5.5384 | 26.5490 | -26.5490 | 37.5460 | 135.0000 |
| 26.3216 | -25.8692 | 5.7016 | 5.9888 | 20.6200 | -31.8580 | 37.9489 | 147.0872 |
| 20.9793 | 22.5824 | 5.0503 | 5.6534 | 15.9290 | 16.9290 | 23.2449 | 43.2568 |
| 18.4018 | 28.1300 | 5.9264 | 5.6556 | 12.4754 | 22.4744 | 25.7048 | 29.0344 |
| 58.7396 | -52.8772 | 5.6426 | 5.5298 | 53.0970 | -58.4070 | 78.9346 | 137.7265 |

TABLE 5. Linear, circular, maps and graphics support statistic computed by QCircularStats plugin.
Circular Statistics by QCircularStats

| Linear Statistics- Modules | Circular Statistics-Azimuts | Maps and graphics |
| :---: | :---: | :---: |
| Number of elements | Number of elements | Module and azimuth distribution |
| Min value | Mean azimuth | Histogram |
| Max value | Mean module | Distribution |
| Range | Circular standard deviation | Points |
| Arithmetic mean | Circular variance | Density Map |
| Mean standard error | Circular dispersal | Q-Q plot |
| Standard deviation | Von Mises parameter | Vectors (only with cartesian data) |
| Variance | Skewness coefficient |  |
| Population standard deviation | Kurtosis coefficient |  |
| Population variance | Tests of uniformity |  |
| Skewness coefficient | Rao test |  |
| Kurtosis coefficient | Rayleigh test |  |

1) Cartesian coordinates: the data is described with fourcolumn format, with the first and second columns defining the origin $\left(x_{o}, y_{o}\right)$ and the remaining two columns the destination $\left(x_{d}, y_{d}\right)$, or the origin node and end node, accordingly.
2) Polar coordinates: the data is represented with twocolumn format, where the former defines the module $d$, and the latter the angle or azimuth $\theta$ of the coordinate.
3) Incremental o difference data: the data is also expressed through two-column format, out of which the first one shows the growth of $x\left(D_{x}\right)$, and the second one shows the growth of $y\left(D_{y}\right)$.
Table 4 provides an example of each supported data format. The software factory settings set the geographic reference system in $0^{\circ}$ or North and the angles in the sexagesimal system, increasing clockwise.

## B. AUTOMATIC COMPUTING OF CIRCULAR STATISTICS

The proposed plugin automatically calculates different parameters of circular statistics which are detailed below. The resulting output data can be categorised into two types, i.e., numerical and graphic. Once the input data (and the path where they are located in case it is a text file) have been selected, the next step is simply to press the 'calculate' option on the key designed for this step (see Fig. 4) and wait for them to be displayed the data on the screen designed for it. Depending on the option checked, different output results will be obtained, i.e., graphical or numerical information.

The output parameters calculated by QCircularStats are briefly summarised in Table 5. For more details on the formula used in its calculation, please see section II, particularly Tables 2 and 3.


FIGURE 5. The study area used for the plugin example (a), its module representation (b), and its azimuths representation (c).

TABLE 6. Linear and circular statistical analysis of winds vectors with output files by $Q C i r c u l a r S t a t s$ plugin. Modules are in meters per second and Azimuths are in degree (except $*$ dimenssionless coefficient).

| Outputs circular statistic analysis | Circular Statistics-Azimuts and Rao and Rayleigh Tests |
| :--- | :--- |
| NUMBER OF ELEMENTS $=1920$ | NUMBER OF ELEMENTS $=1920$ |
| MIN VALUE $=0.10$ | MEAN AZIMUTH $=14.46$ |
| MAX VALUE $=8.01$ | MEAN MODULE $=0.63$ |
| RANGE $=7.91$ | CIRCULAR STANDARD DEVIATION $=0.96$ |
| ARITHMETIC MEAN $=3.89$ | CIRCULAR VARIANCE $=0.37$ |
| MEAN STANDARD ERROR $=0.03$ | CIRCULAR DISPERSAL $=0.93$ |
| STANDARD DEVIATION $=1.52$ | $*$ VON MISES PARAMETER $=1.65$ |
| VARIANCE $=2.30$ | $*$ SKEWNESS COEFFICIENT $=0.09$ |
| $*$ SKEWNESS COEFFICIENT $=-0.08$ | $*$ KURTOSIS COEFFICIENT $=0.70$ |
| $*$ KURTOSIS COEFFICIENT $=-0.13$ | Rao Test: the hypothesis of uniformity is rejected for P $=0.01$ |
|  | Rayleigh Test: P-value for the hypothesis of uniformity $<0.001$ |

It is beyond the scope of this paper to explain in detail each measure and statistic parameter provided by the developed plugin. Nevertheless, it is important to briefly explain the meaning of the graphical results. In QCircularStats plugin includes different features to plot vectors, their modules and the resulting statistics. The proposed QCircularStats provides the following graphics:

- Spatial vector distribution, its azimuth and module.
- Azimuth data distribution, with each azimuth stacked within an unit circle, highlighting the mean azimuth and confidence interval.
- Circular histograms on a polar area diagram, in which sectors are expressed with equal width, and a proportional radius to the relative frequency of azimuths in the corresponding sector.
- A point distribution plot on a polar diagram that represents information.
- Density maps, which plot vector densities, according to a gradient chromatic scale.

These charts and maps are summarised in Table 5 with the abbreviated name presented by the QCircularStats graphical interface. Furthermore, the graphics generated by the proposed plugin are easily exportable and can be edited by third-party software. Section IV provides a practical application example, depicting all the statistical results and possible graphs of the new plugin for circular statistics.

## IV. APPLICATION AND RESULTS OF ONE PRACTICAL CASE: ANALYSIS OF WINDS VECTORS

To evaluate the proposed QCircularstats plugin for QGIS, a real wind dataset for the Atlantic Ocean has been used to conduct the desired circular statistic analysis. The considered wind data has been obtained from the SeaWinds scatterometer, which is installed on the QuikSCAT satellite as part of the NASA Earth Observing System (EOS) for


FIGURE 6. Rose plots and graphics provided by the proposed QCircularStats.
monitoring and acquiring global ocean surface wind vector observations. This instrument gathers daily wind data vectors at the ocean surface. It must be noted that NASA's artificial satellite QuikSCAT was launched in June 20, 1999 via the Titan rocket from the U.S. base at Vandenberg, California, with the mission to measure the speed and direction of winds in the oceans using microwave in order to improve the weather forecast, and study the development of hurricanes and tropical storms.

Particularly, the same target area discussed in [17] has been chosen to be studied, with the aim of assessing and validating the results provided by the statistical analysis conducted through the proposed QCircularStats. In [17], circular statistics were conducted and evaluated with VecStatGraphs $2 D$.
It must be noted that the distribution of the data is unknow. Regarding the data format, considered wind data is represented through angular geographical coordinates, i.e., longitude and latitude, by defining the projections $u$ and $v$ of the wind vector on the X -axis (longitude) and Y -axis (latitude), respectively. These vectors can be used to estimate azimuth,
i.e., the direction that wind blows towards, and vector module, i.e., wind speed, with a $25 \mathrm{~km} \times 25 \mathrm{~km}$ spatial resolution. In this study area, a set of 1920 wind vectors from the Intertropical Convergence Zone (ITCZ) is analysed, due to the clear definition of wind changes. In this example, the analysed area is a $650 \mathrm{~km} \times 2200 \mathrm{~km}$ rectangle located within a $4^{\circ}$ to $10^{\circ} \mathrm{N}$ latitude and $320^{\circ}$ to $340^{\circ} \mathrm{E}$ longitude from 13th August 2002. Fig. 5(a) provides the studied area, whilst Fig. 5(b) and Fig. 5(c) depicts the azimuths and modules of the 1920 samples, respectively.

Table 6 provides a summary with both linear and circular numerical results of the 1920 selected samples. For instance, focusing on the Von Mises parameter, it can be seen that the data are concentrated in the mean. Regarding the Skewness coefficient, the distribution is very close to symmetry, with a slightly longer tail is on the right side of the distribution, whilst the Kurtosis coefficient $(<3)$ indicates a platykurtic distribution with fewer and less extreme outliers in comparison to the normal distribution. Moreover, Fig. 6 depicts the results obtained in a graphical form using the six available visual functions explained in Section III-B. Both ways of presenting the results are complementary.

## v. CONCLUSION

The present paper introduces QCircularStatistic a new plugin to QGIS is an easy-to-use open-source tool allowing the automatic computing of circular statistics and graphic of circular data. It can be concluded that, this work successfully proposes a new open-source plugin for QGIS to conduct circular analysis over remote sensing data in a comprehensive, understandable, and visual way for non-specialist users, as well as other circular statistics data. The plugin provides a simple, easy-to-interpret and easy-to-use GUI, with the capability of loading and interpreting different input formats (cartesian, polar, incremental), offering a high degree of flexibility. Moreover, the GUI provides a wide range of circular statistical operations, the results of which are provided to the user in numerical and visual form. Indeed, obtained results over real sea-wind data gathered by the QuikSCAT instrument demonstrate that the implemented tool properly calculates summary statistics, conducts appropriate hypothesis tests, and provides descriptive pie charts and histograms. The obtained performance, wide functionality, and versatility of the implemented plugin makes it a potential and attractive tool for the Earth Observation scientific community.

As future work, it is proposed to extend the conducted studies on directional data by developing an open-source tool for comprehensive and understandable analysis of spherical data.

## VI. CODE AVAILABILITY SECTION

Name of Software: QCircularStats.
Software license: The GNU General Public License v3.0.
Library Availability: https://github.com/mhaut/PyCircular Stats
Plugin Availability: https://github.com/mhaut/QCircularStats

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[^0]:    The associate editor coordinating the review of this manuscript and approving it for publication was Krishna Kant Singh ${ }^{(1)}$.

[^1]:    ${ }^{1}$ Indeed, the equivalence between concentration parameter $k$ and variance $\sigma^{2}$ is closer to $\sigma^{2}=1-\frac{I_{1} k}{I_{0} k}$, where $I_{1}$ and $I_{0}$ are the first and zeroth order Bessel functions.

[^2]:    ${ }^{2}$ The $U$-distribution calculated by Rao is computationally very complex. Thus, normally tables are used instead of the complete distribution [49].

