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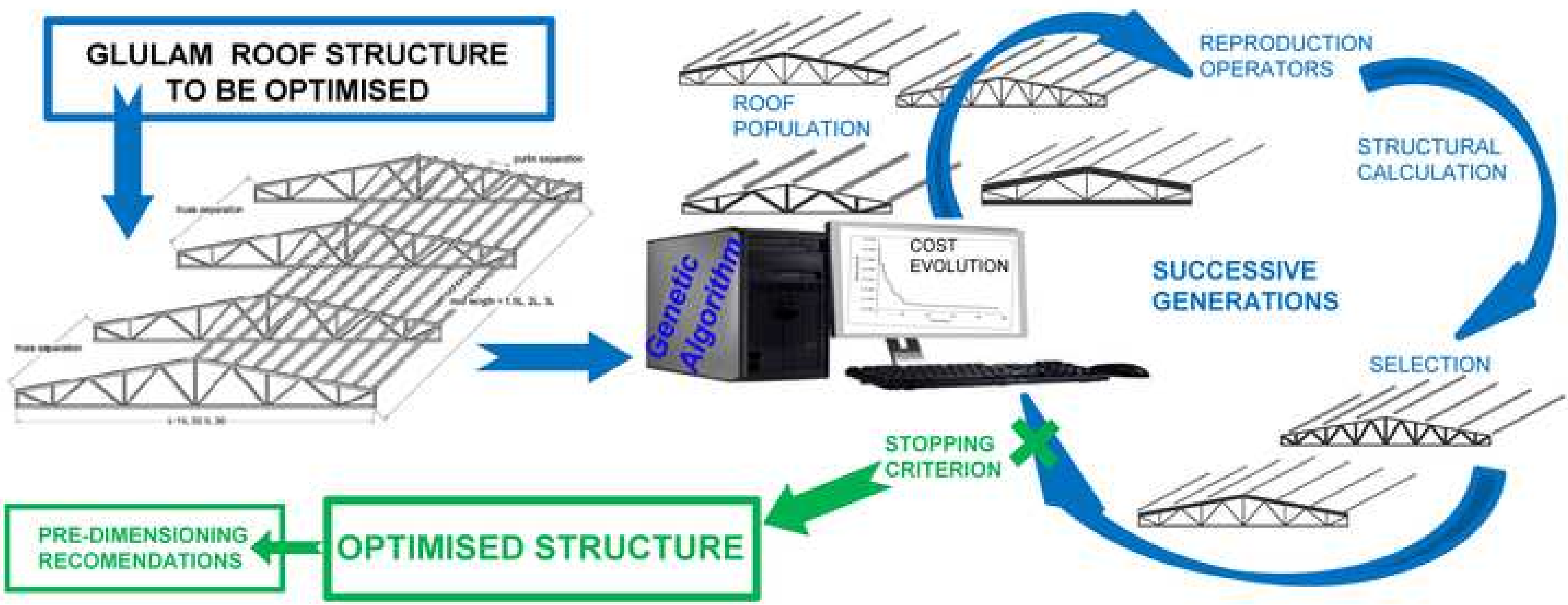

COST OPTIMISATION OF GLUED LAMINATED TIMBER ROOF STRUCTURES USING GENETIC ALGORITHMS

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Abstract

Roof structures comprising of heavy timber trusses and purlins made of glued laminated timber, as well as dowels and metal plates used as mechanical joints, are widely employed, among others, in agro-industrial settings that require large open areas. This paper presents the economic optimisation of such roof structures through the use genetic algorithm models. Two phases of optimisation were carried out: firstly, in two dimensions for a single truss and, then, an entire roof structure in three dimensions. Both models followed a discrete approach, i.e. the optimisation of the cross-section was limited by the characteristics of the commercially-available glulam timber boards, an aspect not yet included in the literature. Therefore, the models allowed the influence of the laminate thickness in the optimisation to be estimated, but also allow comparisons with the continuous cross-section variation found in the literature. Furthermore, the optimisation took into account a range of configurations of trusses, number of joints and separation between trusses and purlins. The genetic algorithms were shown as an efficient optimisation tool for roof glulam structures as a function of the laminate thickness. Among the results obtained, the most cost-effective solutions were those comprised of the fewer number of joints in the trusses and the lowest laminate thickness of those studied. Moreover, the optimal separations between trusses and purlins were also determined. Finally, a simplified method of optimum pre-dimensioning was also proposed.

Keywords: Roof Structures; Timber Trusses; Glulam Timber; Genetic Algorithms; Structural Optimisation.

Symbols

a_1	Spacing, parallel to grain, of fasteners within one row [mm]
a_2	Spacing, perpendicular to grain, between rows of fasteners [mm]
$a_{3,c}$	Distance between fastener and unloaded end [mm]
$a_{3,t}$	Distance between fastener and loaded end [mm]
$a_{4,c}$	Distance between fastener and unloaded edge [mm]
$a_{4,t}$	Distance between fastener and loaded edge [mm]

A_i	Cross-section of member i [mm^2]
A_i^*	Effective cross-section of member i [mm^2]
b	Width of a cross-section [mm]
$ct_{dowel+steel}$	Materials and labour costs per fastener for handling, assembling, drilling, and bolting, including the adjoining steel plates [€ $dowel^{-1}$]
ct_{GL}	Price of the manufactured and embedded timber per m³ [€ m^{-3}]
ct_{hanger}	Materials and manual labour costs for handling and assembling one purlin hanger, 3.75 [€ $hanger^{-1}$];
$c_e(z)$	Wind exposure factor
d	Fastener diameter [mm]
E_{mean}	Mean value of the elastic modulus [N mm^{-2}]
$E_{0.05}$	Fifth percentile of the elastic modulus [N mm^{-2}]
$F(x)$	Modified objective function [€]
$f(x)$	Objective function [€]
$G_j(x)$	Maximum ultimate limit state utilisation ratio in each bar j
h	Height of a cross-section [mm]
ht	Edge depth (i.e. height at the truss supports) [m]
Ht	Greatest depth of the truss (i.e. midpoint height) [m]
j	Number of variables studied
k_{mod}	Modification factor, which takes into account the effect of the duration of the load and the moisture content
K_{ser}	Slip modulus
K_u	Instantaneous slip modulus for ultimate limit states
L	Span of the truss [m]
l_i	Length of member i [mm]
n	Number of members of the upper chord
$n_{a,i}$, $n_{e,i}$	Number of fasteners at the beginning and end of member i
n_{lam}	Number of laminates in a cross-section
N_{dowels}	Total number of dowels in a truss
$N_{trusses}$	Total number of trusses for a "roof individual"
$N_{purlins}$	Total number of purlins for a "roof individual"
$P_j(G_j(x))$	Penalisation of the objective function in accordance with the ultimate limit state [€]
q_b	Wind basic velocity pressure
$S(x)$	Maximum ultimate limit state utilisation ratio
$T(S(x))$	Penalisation of the objective function in accordance with the serviceability limit state [€]
t_s	Steel plate thickness [mm]
V_{GLT}	Volume of glulam for a truss [m³]
V_{GLP}	Volume of glulam for a purlin [m³]

x	Member of the study population
γ_m	Partial safety factor for a material property
ρ_m	Mean density [kg m^{-3}]

Abbreviations

AI	Artificial Intelligence
EC5	Eurocode 5 (CEN EN 1995-1-1: 2016)
GA	Genetic algorithm
GM	General Model
NLP	Nonlinear programming
SLS	Serviceability limit state
ULS	Ultimate limit state
2D	Two dimensions
2DGM	Two dimensions General Model
3D	Three dimensions
3DGM	Three dimensions General Model

1. Introduction

The use of heavy timber trusses is a common practice in construction to achieve large-span roofs that also support the adjustment to a wide variety of shapes as well as offering natural and aesthetic options for interior design. These large trusses are usually comprised of elements made from glued laminated timber and mechanical joints, in most cases, resolved with plates and dowel fasteners. This research work focuses on the structural and cost optimisation of roofs made with such heavy timber trusses and purlins, i.e. this paper aims to find the solution that meets the requirements of functionality and security at the lowest possible cost. The need for the optimisation of these structures arises from the calculation techniques employed by commercial structural calculation programs (i.e. the independent dimensioning of bars and joints that comply with the calculation standards), whereas the economic optimum could be only achieved through general dimensioning algorithms. There are numerous structures comprised of trusses and purlins that comply with the structural standards but the challenge it is to put forward solutions (i.e. calculation schemes) that comply with the standards while representing the lower possible cost. This research addresses the necessary leap between the application of the corresponding structural standards and the global cost-optimisation of a glued laminated timber roof structure.

Optimisation studies of structures are dated back to the 1970's, but in the last two decades artificial intelligence (AI) techniques have been implemented (Houšt', Eliáš, & Miča, 2013; McKinstray, Lim, Tanyimboh, Phan, & Sha, 2015) Among those techniques, genetic algorithms (GA) are one of the most widely recognised and widely employed in the optimisation of steel and concrete structures (Afshari, Hare, & Tesfamariam, 2019; Cazacu & Grama, 2014; Dede, Bekiroğlu, & Ayvaz, 2011; Fernandez, 2014; McKinstray et al., 2015; Park, Chun, & Lee, 2016; Prendes-Gero, Bello-Garcia, del Coz-Diaz, Suarez-Dominguez, & Garcia-Nieto, 2018; Ruo-qiang, Feng-cheng, Wei-jia, Min, & Yang, 2016). However, the optimisation of wooden structures has not experienced the same level of attention from the scientific community. Only a few references focused on timber frames could be found in the literature (S. Šilih, Kravanja, & Premrov, 2010; Simon Šilih, Premrov, & Kravanja, 2005; Topping & Robinson, 1984), which predate the development of the current Eurocodes for timber structures. The application of AI techniques in the study of timber structures was pioneered by the authors, Villar, Vidal, Fernández, & Guaita, (2016), in a paper addressing the optimisation of timber trusses through the programming of genetic algorithms, which resulted in optimisation improvements when compared to earlier methods. However, to the best of the authors' knowledge, there have been no significant contributions in this line of work since then. This research takes a further step in the optimisation of laminated timber structures by extending the optimisation to a three-dimensional roof structure composed of glulam trusses on which purlins are arranged. It is worth mentioning that contrary to the theoretical glulam cross-section approach followed by previous studies (S. Šilih et al., 2010; Simon Šilih et al., 2005; Villar et al., 2016) where a continuous variation of the cross-section dimensions was accepted and any value could be selected, the optimisation method described in this paper was based upon real glulam cross-section constraints, taking into account the laminate thicknesses that are commercially available in the European market. Therefore, the thickness and width of the boards employed to execute the glulam timber have been used which implies that only discrete values of the cross-section dimensions could be selected depending on the thickness and width of the timber boards. In the optimisation of this type of timber structures, attention should be paid to numerous variables that impact on the overall cost (number of joints, member cross-sections, number of fasteners, etc.) as well as determine the structural design through their interaction in the different structural members and at the 3D roof level, i.e. the spacing between trusses and purlins should also be considered. In this regard, GAs have proven to be powerful tools when multiple interacting variables are in play (Villar et al., 2016). Thus, a GA structural optimisation procedure programmed in MATLAB (MathWorks, 2010) was carried out in this paper.

Firstly, a two dimensional (2D) optimisation approach was performed where only the timber trusses were considered in order to compare the results with those obtained by the continuous optimisation implemented in Villar et al. (2016). Subsequently, the optimisation of a complete roof was carried out by arranging purlins on the trusses to obtain a three-dimensional structure (3D). Different truss spans and roof lengths were studied, i.e. the separation between trusses and between purlins was added to the structural optimisation and was incorporated as variables of the research. This allows, in an innovative way, a more realist optimisation as the interaction between trusses, purlins and joints was incorporated in overall cost of the structure.

Therefore, the 3D approach regarded the combined optimisation of separations, cross-sections and joints, which is relevant due to the great importance of joints in the design of heavy timber trusses (Villar-García, Crespo, Moya, & Guaita, 2018; Villar-García, Vidal-López, Crespo, & Guaita, 2019). This experimental design also allowed for a comparison between the 2D and 3D optimisations. Finally, a re-engineering of the results was allowed in order to propose a pre-dimensioning method. In this way, the costs resulting from the optimisation process were considered and the conclusions were used to propose, in an unprecedented manner, a simplified method of optimum pre-dimensioning.

2. Timber roof structures. Structural calculation.

This section addresses the structural calculation of roofs comprised of trusses and purlins. Since the structural calculation of the trusses has already been reported by the authors in a previous paper, only a summary is presented here with a more detailed explanation to be found in Villar et al. (2016).

2.1. Basic parameters

The type of truss employed in this work, originally taken from Blass et al. (1995), was the same used in Villar et al. (2016), which allowed a discrete and a continuous optimisation of the truss cross-sections to be compared. Therefore, duo-pitch roof trusses (Fig. 1) comprising of a horizontal bottom chord and two upper chords all connected by vertical and diagonal intermediate members were assessed. The trusses were classified depending on the number (*n*) of divisions that define the joints in the upper chords as in Villar et al. (2016)..

Figure 1. Truss classification: (a) truss n6; (b) truss n10; (c) truss n14. Taken from Villar et al. (2016)

For three-dimensional optimisation, the roof structure was completed with purlins that perpendicularly connected the trusses. The material of both the trusses and purlins was GL32h glued laminated timber, so the mechanical properties specified in the CEN EN 14080 (2013) standard were adopted. In addition, a roof enclosure without structural function was considered to take into account the load transferred to the structure. Nevertheless, this roof enclosure was not included in the economic optimisation since no variations in cost would result among the different cases of study.

2.2. Ultimate Limit State (ULS) checks

2.2.1. Truss members

The ultimate limit states (ULS) verification of cross-sections and members was performed following the European standard of timber structures CEN EN 1995-1-1:2016 (2016), hereinafter mentioned as Eurocode 5 or EC5.. A detailed explanation could be found in Villar et al. (2016).

2.2.2. Joints

For the structures studied, the joints were defined by dowel fasteners and a steel plate as the central member of a double shear connection. Regarding joint verification, the equations specified in the EC5 (ec. 8.11 secc. 8.2.3 and ec. 8.34, secc. 8.6) were used to assess the structural strength compliance. Nevertheless, it was also necessary to verify the spacing between dowels as per EC5 requirements (Table 8.5, secc. 8.6). A detailed explanation of the joints verification could be found in Villar et al. (2016).

2.2.3. Purlins

According to the EC5, the ULS check of purlins, which were considered simply supported, involved the verification of the strength of the cross-section and the buckling behaviour. Since the separation and length of the purlins were included as parameters in the optimisation approach, the AI algorithm was responsible for applying the corresponding loads as a function of those parameters. However, it was not necessary to perform the calculation of any type of joint for the purlins, and only the presence of support fittings in the trusses was considered to account for the cost effect.

2.3. Serviceability limit state (SLS) checks

The verification of the SLS implied checking the deflection in the middle of the span of the truss and purlin. In the truss deflection, the slippage of its joints was considered since it increases the deformation of the structure as observed by Villar et al. (2016). A value of $l/300$ was selected, which is within the EC5 recommended range of limiting values for the deformation (variable loads).

2.4. Slipping of joints in ULS and SLS

To incorporate the joint slippage in the SLS verification, an effective cross-section A_i^* was considered according to Blass et al. (1995). The effective cross-section reduces the real cross-section A_i of the structure members as a function of the member length, the number of fasteners at both ends of the member, the mean value of the modulus of elasticity (E_{mean}), the slip modulus (K_{ser}) in accordance with EC5, the timber mean density and the fastener diameter, as previously indicated by Villar et al. (2016).

For the ULS verification, a similar effective cross-section expression was used to incorporate the slippage by taking into account the ULS slip modulus, K_u ($K_u = 2/3\ K_{ser}$ as stipulated by EC5), and the 5% value of the modulus of elasticity, $E_{0.05}$.

2.5. Structural design model

In Villar et al. (2016) a structural 2D model was developed for the study of trusses, as a first-order matrix calculation. The authors implemented a general model (GM) that used rigid nodes except for the post and diagonals, which were considered pinned-pinned elements, and considered the structure uniformly loaded. The GM exhibited a greater level of accuracy. Therefore, the GM was used in this research work as it provides a better representation of reality. Figure 2 shows the structural calculation model for both the 2D optimisation, "truss model", and 3D optimisation, "truss and purling model", i.e. simply supported purlins resting on the simply supported trapezoidal trusses.

Fig. 2. Structural calculation models: General truss Model and Purlins Model with boundary conditions.

3. Optimisation parameters

Two types of optimisation were carried out. Firstly, the trusses were studied as an individual element, which resulted in an optimisation in two dimensions, i.e. in the plane of the truss without considering the purlins, based on the general model (2DGM). This approach enables drawing comparisons with the continuous optimisation carried out by Villar et al. (2016) in order to serve both as a validation of the results and an assessment of the influence of introducing a discrete optimisation, which is a more realistic approach than the continuous hypothesis.

The second phase of this study examined the optimisation of trusses on which purlins are arranged, i.e. a spatial structure constituting an entire roof. Therefore, the optimisation was not limited to the sections of the structural elements but also included the separations between the trusses and between the purlins (Fig. 2), which constituted a 3D optimisation (3DGM).

3.1. Timber trusses to be optimised

For the 2DGM optimisation, a truss of 22.5 m span, which corresponds to the one optimised in Villar et al. (2016), was considered to enable further comparisons. In addition, 15 and 30 m span trusses were also studied in the analysis of the entire roof (3DGM optimisation). In this research work, the optimisation was based upon a cross-section discrete approach, which recognises that the cross-section of the laminated timber elements depends on the thickness and width of the boards employed in their manufacture. In this regard, the most commonly used thicknesses employed in the manufacture of glulam timber, which are 35, 40 and 45 mm, were used to obtain the final height of the cross-sections, i.e. the height value was equal to a multiple of one of those values. For the width of the pieces, 80, 100, 110, 130, 140, 160, 180, 200 and 220 mm are usual values (Argüelles, Arriaga, Esteban, Iñíguez, & Argüelles Bustillo, 2013). Nevertheless, it should be noted that width values are greatly dependent on the manufacturer, so values of 90, 120, 190 and 210 mm are also possible. Therefore, the width range examined in this work oscillated between 90 and 220 mm in 10 mm increments.

The geometry of the 22.5 m truss was originally described in Blass et al. (1995): a top chord slope of 10°, raised eaves of ht = 1 m and a maximum depth at the ridge of Ht = 3 m (Fig. 1 and 3). In addition, the top chord was laterally restrained at a 3.8 m separation as in the original truss. For the 15 and 30 m trusses, a scaling of the previously described truss was performed as shown in Fig. 3. By following this approach, the results were not affected by modifications in the structural typology and, therefore, they were comparable.

Fig. 3. Geometry of the trusses considering a span of: 15 m, 22.5 m and 30 m. Example shown for type n10.

Regarding the uniform loads on the trusses, the ones described in Blass et al. (1995) were also adopted: dead loads (2 kN m^{-1}) and snow loads (5 kN m^{-1}). Since the same values were also employed by Villar et al. (2016) and Simon Šilih et al. (2005), a comparison between the different 2D optimisations was possible. The weight of the different structural elements was automatically entered by the algorithm according to the cross-sections examined throughout the optimisation process. Furthermore, it was considered a Service Class 2, a modification factor k_{mod} = 0.9, and a glulam safety factor γ_m= 1.25.

At the joints, the same characteristics as those set in the aforementioned references (Simon Šilih et al., 2005; Villar et al., 2016) were maintained to make the results comparable: diameter of dowel d = 14 mm, thickness of the steel plate t_s = 8 mm, structural steel grade S 235. A modification factor k_{mod} = 0.9 for short term load and safety factor γ_m= 1.3 were used to calculate the dowels design load carrying capacity. Then, the algorithm calculated and optimised the number of dowels in the joints. Since a minimum height of the connected members is required depending on the number and spacing between dowels, which has a clear implication in the minimum number of laminates needed (Fig. 4), a 14 mm diameter of dowel was considered, which is the minimum value usually employed in this type of trusses.

Fig. 4. Example of joint with dowel fasteners and minimum spacings following EC5.

3.2. Roof structure to be optimised

The 3D roof structure was comprised by the glulam trusses described in the previous section and the purlins arranged between the trusses, which constitutes a common structural solution for large open surfaces, e.g. livestock facilities, agro-industrial warehouses or any other building in rural environments.

Regarding the dimensions of the roof, different widths were optimised (3DGM) corresponding to the span of the trusses: 22.5 m, 15 m and 30 m, which are normal values for this type of trusses. The roof structure was also optimised for different lengths depending on the span: 1.5, 2 and 3 times the truss span were examined in order to assess the influence of the amount of material in the trusses for the different lengths (Fig. 5). The maximum length was limited to 3 times the span of the truss since previous tests with greater lengths resulted in a practically constant cost per m^2 from this point forward. The purlins were made of the same type glulam as the one employed for the trusses, and the same assumptions about the laminate thickness and widths indicated for the manufacture of the trusses were also considered for the purlins. They were arranged as simply supported members between two adjacent trusses. It is to be noted that, for the same study case, all elements of the structure were implemented with the same laminate thickness.

Fig. 5. Roof structure to be optimised, example for truss type n10.

The 3DGM optimisation included everything indicated for the trusses but also included the optimisation of the cross-sections, length and arrangement of the purlins. The length of the purlins was considered a variable, which enabled the algorithm to select the optimum separation between trusses to minimise the cost. Likewise, the variation of the lateral separation between purlins was also allowed. For instance, the use of a roof cover executed with wood sandwich panel with thermal insulation would allow a separation between purlins ranging from 625 mm to 1250 mm, as long as the roof load allows such variation. The top chord was considered laterally restrained at a length value equal to a multiple of the separation between purlins, i.e. two times the separation in the 15 m span truss, three times in the 22.5 m truss, and four times in the 30 m truss.

The surface loads applied in the 3DGM were similar to those indicated for the 2DGM optimisation. However, in this case, the loads, were expressed in kN per m^2 so they could be directly applied to the purlins depending on the different separation between purlins and between trusses. Therefore, the following loads per m^2 were considered: a dead load of 0.45 kN m^{-2} (it should be noted that this value does not account for the weight of the purlins, which was introduced according to the cross-section resulting from each step of the optimisation process) and a snow load of 1.25 kN m^{-2}, which is an usual value in Europe. In this way, the value of the loads in the 2DGM model and in the works of Blass et al. (1995), Simon Šilih et al. (2005) and Villar et al. (2016) would imply a separation between trusses of 4 m, which constitutes an usual value in the practice. Nonetheless, such value was not specified in none of those works as the optimisations carried out were at the truss level, i.e. 2D. Furthermore, the wind effects were only considered for the roof surface, since the side raised eaves and walls were regarded as self-supporting without transmission of horizontal loads to the trusses. The wind load was determined according to the European standard Eurocode 1, CEN EN 1194-1-4:2018 (2018), by taking into account a wind basic velocity pressure q_b = 0.45 kN m^{-2}, a terrain category II, an exposure factor $c_e(z)$ = 2.1, and following the Eurocode 1 sections 4.2, 4.3, 4.5 and 7.2 to apply the pressure coefficients for buildings.

Table 1 summarises the optimisation parameters presented in this section.

4. The Genetic algorithm optimisation

The fundamentals of the genetic algorithms applied to timber structures have been detailed in the authors' previous work and, thus, a more detailed explanation can be found in Villar et al. (2016). So, this section only addresses the adaptation of the genetic algorithms to this research work.

4.1. Individuals

Since two optimizations were made, it was required to establish two types of individuals: "trusses" for the 2DGM optimisation, and "roofs", i.e. the roof structure comprised of trusses and purlins, for the 3DGM optimisation.

4.1.1. Individuals for the 2DGM optimisation

Meanwhile the chromosomes encode the variables involved in the design (fasteners, cross-sections, etc.) that define the individuals of the genetic algorithm, each individual is a solution to the structural calculation.

The truss members were grouped for the same cross-section into three sets: top chord, bottom chord, and intermediate (posts and diagonals) members according to their structural performance. For the discrete optimisation, it was imposed that all elements of a “truss” individual were required to have the same laminate thickness, i.e. 35, 40 or 45 mm as previously indicated. Furthermore, the individuals were coded according to the number of laminates glued to achieve their final height (h) and their width (b) according to the discrete values indicated in section 3.1. By following this discrete approach, more realistic results than those obtained in the continuous optimization carried out by Simon Šilih et al. (2005) and Villar et al. (2016) are to be expected.

Regarding the joints, the number of fasteners at the ends of a bar was allowed to vary between 1 and 100. The optimal solutions would be those with the minimum number of dowels and minimum steel plate surface while complying with the strength criteria. Thus, the minimum required area for dowels placement also influenced the number of laminates needed in the cross-section for the different thicknesses.

4.1.2. Individuals for the 3DGM optimisation

In the 3D optimisation, the chromosomes encoded both the dimensional characteristics of the trusses, joints and purlins as well as their spatial arrangement and number. Meanwhile each individual included both the previously described trusses and the purlins used to connect them. A discrete optimisation was carried out and a constraint of equal laminate thickness for all elements comprising the structure was imposed, which is plausible since the entire structure would come from the same manufacturer. In the optimisation of a surface defined by the span of the trusses and the length of the building, the individual "roof" was constituted by a specific number of trusses and purlins depending on their separation values, which were parameters added to the optimisation. It is worth mentioning that, in this case, a continuous variation of the separation values was allowed.

4.2. The population

The population size is important for the proper operation of the algorithm. Small populations may impede the GA to reach the entire search space, whereas large populations may involve high computational costs (Yang, 2014). In this work, two different populations were examined according to the optimisation performed: the population of "trusses" for the 2DGM optimisation and the population of "roofs" for the 3DGM optimisation.

For similar structural optimisations, several authors have employed populations ranging from 60 to 250 individuals (Cazacu & Grama, 2014; Dede et al., 2011; Prendes Gero, García, & del Coz Díaz, 2006; Toğan & Daloğlu, 2006, 2008; Wang & Ohmori, 2013; Yu, Li, Jia, Zhang, & Wang, 2015), but there have been instances in which up to 500 individuals have been examined (Dede et al., 2011; Talaslioglu, 2009; Wang & Ohmori, 2013). Regarding the population of "trusses", an initial population of 300 individuals was considered in the previous work (Villar et al., 2016), the discrete optimisation proposed in this paper entailed a finite number of possible cross-sections which limits the need for large populations. Therefore, tests were conducted to reduce the 300 individuals and, thus, to increase computational efficiency without compromising the exploration of the optimum. Finally, a 2DGM population consisting of 150 individuals was considered throughout the entire optimisation, which was a similar value to the one employed by Prendes-Gero et al. (2018, 2006) in the optimisation of concrete steel profiles. The selected number of individuals resulted in a total runtime between 5 and 15 min running MATLAB as interpreted language and saving the results in a computer Intel(R) Core(TM) i7 CPU 2.40GHz, 6.00GB RAM. The runtime depended on number of truss members, but the time was around a 40-50 % of the time employed to reach the optimum in a population of 300 individuals. From 150 individuals onwards, the number of generations needed to reach the optimum is stabilised but the number of evaluations of the objective function increases, which also rises the computational consumption without offering improvements.

For the population of "roofs", a sensitivity study was carried out to determine the number of individuals needed to reach the global optimum solution, taking into account the new variables (purlins cross-sections, separations between trusses and between purlins…), but without an excessive computational cost. Given the lack of previous references in this regard, the sensitivity study considered populations between 150 and 500 individuals. Ultimately, it was found that a population comprised of 330 individuals was required to reach the optimum, which required runtimes between 25 and 45 min depending on the number of truss members as well as the different parameters considered in each case. The selected number of individuals decreased the runtime around 30-40 % compared to the initial population (i.e. 500 individuals). From 330 individuals onwards, the same rising behaviour identified in the 2D model was also noticed, which advised against the increase of the population.

4.3. The objective function

In a genetic algorithm, the objective function or fitness function collects the variables that intervene in the design in order to propose a value, such as volume of material, cost, etc., that expresses the effectiveness of the design. Therefore, the optimal solution is reached for the minimum value of the fitness function. In this work, two objective functions were defined: one for the optimisation of the trusses (2DGM) and other for the optimization of an entire roof (3DGM).

The objective function reflected both the cost of timber and the production cost of all the joints. For the 3DGM optimisation, the new costs associated to the purlins and the purlins hangers that were used to arrange the purlins on the trusses were also included. In addition, variables such as the span, number of trusses and purlins, separation between purlins were also considered to correctly determine the cost and, thus, the roof optimisation. Eq. (1) illustrates the objective function employed in the 3DGM optimisation:

$$f(x)=(ct_{GL}\cdot V_{GLT} + ct_{dowel+steel}\cdot N_{dowels}) \text{ x } N_{trusses} + (ct_{GL}\cdot V_{GLP} + 2 \text{ x } ct_{hanger}) \text{ x } N_{purlins} \quad (1)$$

where:

$f(x)$ manufacturing (material and labour) costs function of the structure [€];

ct_{GL} price of the manufactured and embedded timber material per m³, 900 [€ m^{-3}];

V_{GLT} volume of glulam in a truss [m³];

$ct_{dowel+steel}$ material cost and the manual labour costs per dowel for handling, assembling, drilling and bolting, including the adjoining steel plate, 2.5 [€ $dowel^{-1}$];

N_{dowels} total number of dowels.

$N_{trusses}$ total number of trusses for a "roof" individual;

V_{GLP} volume of glulam for a purlin [m³];

ct_{hanger} material cost and the manual labour costs for handling and assembling one purlin hanger, 3.75 [€ hanger $^{-1}$];

$N_{purlins}$ total number of purlins for a "roof" individual.

In the optimisation of an entire roof, the number of trusses $N_{trusses}$ and the number of purlins $N_{purlins}$ were obtained by the algorithm in each case once the separation between trusses and between purlins were defined. It is worth mentioning that the objective function employed in the 2DGM optimisation coincides with the first addend of Eq. 1.

In order to compare the discrete and continuous optimisation approaches, the costs used is the previous paper were also maintained for this work as they were considered to be still valid.

In each generation, the haphazard creation of the population results in individuals who do not meet the restrictions imposed by the calculation rules. The restricted problem is converted to an unrestricted one by incorporating a penalty inside the objective function (Yang, 2014). This penalisation could be conditional on the level of infringement of the calculation rules. The modified objective function (Eq. 2) applied in the optimisation of individual “trusses” (2DGM) is the same as the one defined by Villar et al. (2016):

$$\mathrm{F(x)} = \mathrm{f}(x) + \sum_j [P_j(G_j(x))] + T(S(x)) \quad (2)$$

$F(x)$ modified objective function [€];

$f(x)$ cost objective function [€];

x individual of the study population;

j number of variable (member) studied;

$P_j(G_j(x))$ cost penalisation of the structure according to ULS as a function of $G_j(x)$ [€];

$G_j(x)$ maximum utilisation ratio produced in each member j of individual x in the ULS including checking fasteners. The utilisation ratio is the degree of compliance of the ULS design conditions in a section – including the check of the member's buckling instability. Higher values of $P_j\ (G_j(x))$ would imply that constraints are not satisfied, $G_j(x) > 1$ and $G_j(x) < 1$, so the algorithm is required to adjust to the compliance limit.

$T(S(x))$ penalisation according to SLS as a function of $S(x)$ [€], it is a single penalty for the whole truss. It is introduced so that $T(S(x))$ is calculated as a function of $S(x)$ = vertical deformation / deformation limit. Higher values of $T(S(x))$ would imply that the constraints are not satisfied, $S(x) > 1$ and $S(x) < 1$, so the algorithm is required to adjust to the compliance limit.

More details regarding $P_j\ (G_j(x))$ *and* $T(S(x))$ could be found in Villar et al. (2016)

For the 3DGM optimisation, the modified objective function was similar to Eq. (2) but also included two new terms to take into account the penalisation of the purlins according to the ULS and SLS compliance.

This modified objective function was used to rank by fitness the individuals of a generation.

4.4. The reproduction operators

Since the fundamentals of these operators have already been exposed by the authors (Villar et al., 2016), this section addresses only the particularities pertaining to the optimisation carried out in this paper, whereas more details on each operator could be found in the aforementioned paper. To define the magnitude of the algorithm operators, a sensitivity analysis was previously implemented by testing the set of values found in the literature review. Finally, the selected values were those that led to improve the efficiency achieved by the algorithm, and to guarantee that the algorithm would reach the optimal solution.

4.4.1. The selection and cross-over operators

The roulette-wheel selection operator was used. This operator is characterized by a proportionality to the fitness selection, which implies that more opportunities for reproduction are given to the fittest individuals. In addition, the crossover operator ensured the transmission of the characters among the best candidates. This research work employed a two-point crossover, i.e. two points on the parents' chromosomes are selected and the sections between those points are exchanged to create the offspring chromosomes, since it has been demonstrated to be effective in the optimisation of the trusses' cross-sections (Villar et al. 2016). The crossover probability defines the population percentage that will take part in the crossover. For the optimisation of individual trusses (2DGM), the previous sensitivity analysis indicated that a crossover probability of 80% rendered the lowest result of the fitness function. For the optimisation of entire roof (3DGM), the crossover percentage also remained effective at the same value. Therefore, a 80% crossover probability was established for both optimisations, which is in line with the values found in other works (Cazacu & Grama, 2014; Fernandez, 2014; Prendes-Gero et al., 2018; Villar et al., 2016).

4.4.2. The elitism operator

The elitism operator prevents the loss of the fittest individuals in subsequent generations, which accelerates the optimisation. The elitism operator is established as a percentage of the total. For the 2DGM optimisation of individual trusses, a 7 % elitism percentage was selected after conducting a sensitivity study, which was slightly lower than the one used in Villar et al. (2016). This discrepancy could be attributed to the lower possibility of variation within the population given the discrete analysis of the cross-sections. For the entire roof 3DGM optimisation, a 10 % elitism percentage was selected after conducting a sensitivity study. Both values were in the range of those found in the literature review (Cazacu & Grama, 2014; Fernandez, 2014; McKinstray et al., 2015; Prendes-Gero et al., 2018).

4.4.3. The mutation operator

The mutation operator allows the algorithm to escape from local minima. This operator alters components in the chromosomes of some individuals of the population. The mutation positions in the chromosomes were selected at random. A previous sensitivity study showed that a percentage of mutation of 1% (Villar et al., 2016) was the most suitable for the optimisation of both trusses and the entire roof. This value was close to those employed by other authors (Fernandez, 2014; Prendes-Gero et al., 2018; Prendes Gero et al., 2006).

4.4.4. The stopping criterion

To conclude the optimisation process, the possible iterations were limited to 150 generations. However, it should be noted that a great number of calculations were finished by convergence, which is achieved through 50 generations without improvement and a 10^{-6} tolerance.

5. Results and discussion

5.1. Optimisation of a single truss (2DGM). Comparison of the discrete and continuous optimisation

Table 2 shows the results obtained for the 22.5 m span truss in terms of volume of timber, resulting cross-section, total number of dowels, instantaneous deflection including the slip of the joints in the mid-span and cost. The table includes the results of the three typologies of trusses (*n*6 *n*10 and *n*14) optimised through the discrete cross-section approach for each laminate thickness, as well as to those obtained in the continuous optimisation carried out by Villar et al. (2016). By comparing both optimisation approaches, an increase in both the volume of timber and the cost was observed when the discrete optimisation was applied. Nonetheless, it was observed that the cross-sections obtained for the discrete and continuous optimisation were consistent and reasonable for all cases and, thus, similar height to width ratios and total dimensions were obtained.

The observation of all the results (Table 2) led to the following comments:

(i) Generally, the discrete optimisation adjusted the cross-section through width increments to avoid increasing the cost by adding a higher number of laminates, which represents a greater increase in cross-section area. In some cases, since a minimum cross-section height is required to comply with the minimum spacing between the dowels and between the dowels and the edges (Fig.4), the algorithm selected the number of laminates that ensured the minimum height and, then, adjusted the width in 10 mm increments according to the needs of the cross-section. This approach seems a more realistic calculation than that of the continuous optimisation, in which the algorithm adjusts the height at the exact value required by the calculation and, at the same time, the minimum width is also maintained. The difference is especially noticeable for intermediate members, the discrete optimisation showed width (b) increases before increasing the height by adding one more laminate.

(ii) The truss comprised of fewer number of elements (n6) was the most economical for all laminate thicknesses, as well as required lower volumes of timber. This result was also observed for the continuous optimisation carried out by Villar et al. (2016).

(iii) The consideration of the laminate thickness implied an increase, on average, of 5.20 % in the volume of timber employed. Thus, the discrete approach triggered an average cost increase of 2.59 % and a maximum rise of 6.20 % for the truss *n*14. This cost increase was a consequence of the increase in volume of timber since the cost of the joints was not altered, on the contrary, the number of dowels was slightly reduced as a result of the new cross-sections, Fig. 6.

Fig. 6. Total cost (€) of a 22.5 m span truss depending on the laminate thickness and truss typology. Comparison between the discrete approach and the continuous optimisation (C-OPT from Villar et al., 2016)

(iv) The laminate thickness influenced the cost and, in general, the 35 mm thickness was the most economical solution. Although, 40 and 45 mm thicknesses resulted in greater costs, no clear tendency between the cost increase and the thickness was observed. Moreover, within each truss typology (*n*6, 10, 14), the cost increase as a function of the thickness was variable. Nevertheless, the 35 mm thickness exhibited the lowest increase when the discrete approach was compared to continuous optimisation (Fig. 6), which implies a better capacity of adaptation to the continuous optimisation of the cross-section. For the most economical truss, *n*6, and the smaller thickness, 35mm, the discrete approach resulted in a cost increase of 1.12 % compared to continuous optimization, the cross-section and the number of dowels were practically similar, whereas a slightly higher volume of timber was required. These results indicated that the discretisation of the cross-section to commercial thickness values may not imply a large cost rise.

(v) For the discrete optimisation, the larger cross-sections and the reduced number of dowels resulted in a slightly lower deflection values due to a diminished slippage of the joints. In addition, in general, by increasing the laminate thickness used, deflections were reduced as a consequence of the larger cross-sections at the same time greater laminate thicknesses reduces the ability to approach the structural optimum.

(vi) Regarding the height to width ratios, values close to the unit (0.9 on average) were noticed for the top chords, while the intermediate members and the bottom chord tended to rectangular cross-sections and average values of 1.59 and 1.70 were observed respectively. The top chords, that were subjected to compression, required greater width values to resist the compressive buckling in the plane perpendicular to the truss, what was not a limiting issue for the bottom chords or most of the intermediate members. Such differences were also stated in the continuous optimisation and similar height to width ratios were reported (Villar et al., 2016). However, for the intermediate members, the rise of the cross-section area through the width increase instead of the height increase by the addition of a new board prevented ratio values as high as in the continuous optimisation. There was no thickness value that closely approximated the height to width continuous ratios, i.e. a different thickness was best suited for a different member type. Although, it should be pointed out that 35 and 40 mm thickness had the closest fits.

No direct comparison between the discrete results and the nonlinear programming (NLP) continuous optimisation carried out by Simon Šilih et al. (2005) was possible since different standards were followed, i.e. versions and drafts previous to the current Eurocode 5 (CEN EN 1995-1-1:2016, 2016) and to the current material characterization norm (CEN EN 14080, 2013) were used in the NLP optimisation. Nonetheless, an indirect connection could be performed by comparing the NLP vs the GA carried out by Villar et al. (2016). The latter indicated cost improvements of 4.25 %, 7.49 %, and 13.44 % for *n*6, *n*10 and *n*14 trusses, respectively. Therefore, in spite of the cost increase previously indicated between the discrete and to continuous optimisation, the discrete approach still maintained a margin of cost reduction compared to the NLP optimisation.

5.2. Optimisation of an entire roof (3DGM)

5.2.1. Truss types for 3DGM optimisation

The results of the 2DGM indicated that, for any laminate thickness studied, trusses made of fewer members resulted in more economical solutions. In order to verify this fact for the entire roof, a 3DGM optimisation was carried out for the intermediate thickness, 40 mm, and the three types: *n*6 *n*10 *n*14. Similarly, trusses comprised of fewer members were the most economical alternatives. Fig. 7 illustrates the cost per m^2 for the 22.5 m span truss and the three roof lengths considered: 1.5, 2 and 3 times the span of the truss.

Fig. 7. Cost (€ m^{-2}) for the 40 mm laminate thickness and 22.5m span (L) truss depending on the truss typology and roof length (multiple of L).

Based on the results, typology *n*14 was disregarded due to its higher costs and the subsequent 3D economic optimisation was focused on the *n*6 and *n*10 configurations of the 22.5 m truss for the different laminate thicknesses. It was decided to address both the *n*6 and *n*10 truss, to take into account the possibility that the joined consideration of the truss typology and the thickness variation could alter the previous cost findings when the optimization of the entire roof was considered.

Regarding the 15 m truss, the *n*14 typology was also not studied since the resulting dimensions of such configuration could not be considered as a heavy timber truss, which is the structure examined in this paper. Thus, the initial study was carried out for the *n*6 and *n*10 configurations of the 15 m truss and the intermediate board thickness, 40 mm. The results corroborated again the previous findings, the most economical option lied in the use of the truss comprised of fewer members (Fig. 8). Therefore, the subsequent optimisation study was only performed for the *n*6 typology of the 15 m span as a function of the board thickness (35, 40 and 45 mm).

Fig. 8. Cost (€ m^{-2}) for 40 mm laminate thickness and 15m span (L) truss depending on the truss typology and roof length (multiple of L) with 40 mm laminate thickness.

For the 30 m span truss, a similar procedure was followed. In this case, typology n6 was initially discarded due to lack of structural sense, since it would originate excessively long and slender truss members. Thus, the initial comparison was performed for the n10 and n14 configurations of the 30 m truss and the intermediate thickness, 40 mm. In this instance (Fig. 9), the results advised to carry out the optimisation study on the n10 typology of the 30 m span as a function of the board thickness (35, 40 and 45 mm).

Fig. 9. Cost (€m^{-2}) for 40 mm thickness and 30m span (L) truss depending on the truss typology and roof length (multiple of L).

It is worth mentioning, that the selection of the trusses for the different optimisations was not based solely on the aforementioned figures, as the decision was also supported by the numerical results illustrated in Tables 3, 4, 5, 6 and 7, which constitute the findings of the optimisation carried out in this research work and would be further analysed in section 5.2.2.

5.2.2. Discussion of 3D optimisation results

In this section, the results of the optimisations carried out for the different truss typologies, truss spans, roof lengths and laminate thickness studied are analysed. Tables 3, 4 ,5 ,6 and 7 show the resulting costs (total cost, cost of the trusses and cost of the roof structure per m^2) once each case was optimised. In addition, the tables also indicate the structural characteristics of the solutions reached in each case: cross-sections of all members, total m^3 of timber, total number of dowels, separation of trusses and purlins, as well as the instantaneous deflection at mid-span when the slip of the joints was considered. Particularly, Tables 3, 4 and 5 illustrate the characteristics of the structural optimum solutions depending on truss span and typology, roof length and laminate thickness. Meanwhile, Tables 6 and 7 collect the non-optimal typologies, which were employed in the selection of the truss typologies to be used in the final 3D optimisations (section 5.2.1).

As an example of the optimisation process, Fig. 10 illustrates the evolution of the fitness function of the 3DGM, i.e. the optimisation of the entire roof structure characterised by a *n*10 truss typology, a 30 m span truss, a roof length equal to 3 times the span (3 x L) and a 45 mm board thickness. The decrease of the total cost towards the minimum value occurred more steeply in the first generations. The optimal result was achieved after 46 iterations and, in this case, the process concluded by convergence, Haupt & Haupt (2004) qualified this kind of behaviour as excellent. Cazacu & Grama (2014), Wang & Ohmori (2013) and Ruo-qiang et al., (2016), who performed GA optimisations, also reported a similar behaviour of the objective function. Conversely, a fewer number of generations were necessary for convergence compared to the continuous optimisation in Villar et al. (2016). The reduction in the number of generations (around 50%) was a consequence of the width and height constraints of the possible cross-sections, i.e. due to the discrete variation, which was especially significant for the 45 mm thickness.

Fig. 10. Evolution of the fitness function of an entire rood characterised by a n10 truss typology, a 30 m span truss, a roof length of 3 times the span (3 × L) and a 45 mm laminate board thickness.

Finally, in order to further the discussion, the analysis of the results displayed in Tables 3, 4, 5, 6 and 7 prompted the remarks that could be observed in the following subsections.

5.2.2.1. Influence of truss span or roof width.

For the same truss span, trusses comprised of fewer members were reported as the most economical solution, which concurs with results obtained for the 2DGM optimisation. As the truss type "*n*" increased, the truss contained a higher number of members and joints and, thus, the cost of the structure also increased accordingly to the higher volume of timber and number of joints per truss. Similarly, as the truss span increased, the surge in cost per m^2 was accompanied by the increase in the cost percentage of the trusses on the overall total cost, as a consequence of the greater volume of timber needed to cover the larger span.

Regarding to the cross-sections, for the 15 m span truss (Table 3), the bottom chord and the intermediate members tended to be optimised at the minimum width (i.e. 90 mm) and the minimum height required due to dowel spacing according to EC5, i.e. the minimum number of laminates that met the aforementioned limit. In the upper chords, the width of the cross-sections was increased while the height remained at the minimum number of laminates required by the arrangement of dowels, i.e. the algorithm obtained the optimum by increasing the width in 10 mm increments up to 220 mm before including one more laminate to the cross-section, which resulted in the use of greater volumes of timber. Additionally, the width increases had a positive effect preventing the buckling in the perpendicular direction to the truss.

As expected, 22.5 m span trusses also required greater cross-section areas (Table 4). Whenever possible, the algorithm proposed a width increment instead of the increase in the number of laminates. For the intermediate members and the bottom chord, the final height often was determined by the minimum height required by the dowel spacing requirement at the expense of greater width values. In fact, a similar behaviour was noticed up to the 30 m span truss and, for most cases, no significant increases in the height of the cross-sections were observed (Table 5). Conversely, for the upper chord, the final height of the cross-section was usually higher than the minimum height required by dowel spacing. Nonetheless, for some cases (*n*10 and *n*14 configurations of 22.5 m trusses), the final height coincided with the minimum requirement at values of 135, 140 and 160 mm, at the expense of higher height to width ratios (*h/b*), around 2.

For the 30 m span trusses, larger and quasi-quadrangular cross-sections were required in the top chords.

For all cases, the deflection increased with the span but did not exceeded the SLS limit. It is worth mentioning that, for a same truss span, the deflection increased for larger values of truss type “*n*” due to the slippage effect of a greater number of joints.

5.2.2.2. Truss separation

The values reported for truss separation were among those commonly used in the practice (between 4 and 4.5 m). This behaviour was especially noticed for trusses comprised of fewer members and roof length equal to three times the truss span. For instance, a separation between trusses of 4 m was proposed when a 40 mm laminate thickness was employed, whereas a 4.5 m separation was reported for those trusses made of 35 and 45 mm laminates. Nonetheless, some exceptions were observed for *n*14 trusses and recommendations for a 5 m truss separation appeared as the algorithm attempted to reduce the volume of timber and, so, decrease the overall cost by reducing the total number of trusses. Thus, for an increasing “*n*” type, the truss spacing also tended to rise to counteract the volume increase added by a new truss, which was especially significant from *n*10 to *n*14 type. The variation of the truss separation hardly modified the cross-sections of the truss members. Nonetheless, separation values of 5 m caused an increase in upper chord cross-sections while barely affected the intermediate members or the bottom chord.

A similar behaviour was observed for the purlins, their cross-section hardly varied with the separation between trusses and the purlins span, except for those cases resulting in greater truss separations, 5 m, that also required greater height values in the cross-sections. For most cases, the cross-sections of the purlins were optimised for the minimum commercially-available width (90 mm), whereas the remaining purlins required a 100 mm width. In any case, the height to width ratios were very similar, ranging between 1.8 and 2. Nonetheless, some outliers were reported for trusses of higher "*n*" and greater separation (height to width ratio =2.2.), or for a truss separation lower than 4 m (height to width ratio =1.5).

5.2.2.3. Purlin separation

Regarding the separation between purlins, the algorithm always found the optimum at the maximum admissible separation, i.e.1,250 mm. Since the addition of each new line of purlins results in an important increase of timber, the algorithm always proposed the minimum number of purlins to achieve the optimisation of the entire structure. The behaviour exhibited by the algorithm corresponds to the usual procedure in the roof construction, i.e. to separate the purlins as much as its allowed by the load and the roof cover.

5.2.2.4. Influence of roof length

As the length of the roof increased, the cost per m^2 decreased slightly. Furthermore, the influence of the cost of the trusses on the overall structure also was reduced. However, this behaviour was less apparent when the 40 mm laminate thickness was employed.

The purlins cross-sections remained constant with the roof length increase due to the small variation in the spacing between trusses. Nonetheless, as it has been already indicated, a 10 mm increase in the width of the purlins was observed when the truss separation reached 5 m. A similar behaviour was noticed for the truss members; small differences were observed for the cross-sections of the upper chords due to the variations in trusses separation. Conversely, in general, the cross-sections of the bottom chord and the intermediates members remained constant.

Finally, it was observed that, in a roof with a length equal to three times the truss span, the cost-effect of the initial and final trusses into the overall structure was diluted, which could be considered equivalent to study an infinite roof length structure.

5.2.2.5. Regarding to the laminate thickness

The cost of the structure was affected by the selection of laminate thickness necessary to fit the optimal theoretical cross-section, which was calculated according to both the structural and dowel spacing requirements. In general, the results obtained for the 3DGM spatial optimisation confirmed the findings previously noticed in the 2DGM optimisation.

Firstly, the variation in the laminate thickness did not affect the fact that the most cost-effective solution is achieved by employing trusses comprised of fewer members.

For the same truss type and span, the laminate thickness modified the cost, and the 35 mm thickness resulted in the most economical alternative, followed by the 40 and 45 mm thickness. Although no clear trend was apparent, the 40 mm laminate resulted the less economical thickness, especially when it was employed for the 15 and 30 m span trusses. Thus, the 35 mm thickness exhibited the best fit to the theoretical cross-section and the dowels spacing requirements. Figure 11 shows the cost of for the 15 m span truss with the optimal typology, n6, according to Table 3. The timber volume was a main factor on this behaviour followed by the influence of the number of dowels.

Fig. 11. Total cost (€) of a 15 m span truss with optimal type n6

Regarding the cross-sections, variations in the laminate thickness caused the following behaviours:

- In the upper chord, for the same truss span and typology, the tendency observed pointed to the use of a 35 mm laminate thickness to achieve lower cross-sections. However, no similar trend was noticed for the other two thicknesses. In general, for roof lengths equal to three times the truss span, the optimum was obtained with 4 laminates for the 15 m truss and 5 - 6 laminates for the 22.5 and 30 m trusses, while the width of the cross-section increased progressively with the span. In general, the highest height to width ratios were obtained for the 40 mm thickness, whereas there was no clear trend for the other two thicknesses.

- For the bottom chord, the increase in laminate thickness tended to rise the area of the cross-section obtained through optimisation. Similarly, the height to width ratios also increased for the same truss span and type when greater thicknesses were employed. In general, for roof lengths equal to three times the truss span, the optimum was obtained with 4 laminates and a width of 90 mm for the 15 m truss, 90-100 mm for the 22.5 m truss and 120-130 mm for the 30 m truss.
- For the intermediate members, the largest cross-sections were obtained for the 40 mm thickness, whereas similar cross-section values and height to width ratios were obtained when the 35 and 45 mm laminates were employed. In general, the optimal solution was reached, for roof lengths equal to three times the truss span, with a height of 3 laminates for the 45 mm thickness and 4 laminates for 35 and 40 mm thickness, while the optimum width followed a similar trend to that of the bottom chord.

The aforementioned tendencies indicated that the cross-sections and height to width ratios of the bottom chord could be attributed to the algorithm that determined the width and the number of laminates according to the structural requirements and no modifications were needed thereafter since there was no need to stabilise the bottom chord against buckling. However, the upper chords and intermediate members could be subjected to buckling in the perpendicular direction to the truss span. Thus, the algorithm had to adjust the width to avoid buckling and, at the same time, the height through the number of laminates according to the board thickness. In this regard, it was noticed that the 35 mm thickness resulted in the best fit to the minimum cross-section, whereas the 45 mm and, especially, the 40 mm thickness exhibited more difficulties to fit the calculated cross-section without exceeding the optimum. It should also be mentioned that the determination of the cross-section in lower span trusses, was more influenced by the minimum height necessary to comply with the dowel spacing requirement than those of greater span, whose cross-section requirements easily exceeded the limit imposed by the dowel spacing.

Furthermore, for trusses comprising of fewer members (i.e. optimal trusses) and roof lengths equal to three times the truss span, the spacing between trusses (i.e. the length of purlins) tended to 4.5 m for the 35 and 40 mm thicknesses, and 4 m for the 40 mm thickness, which was in line with the previous finding that the 40 mm laminate offered the worse economic results.

Regarding to the purlins, the thickness variation did not cause the algorithm to modify the width cross-section, but the height was adjusted as much as possible. Although the thickness increase rose the final cross-sections, no clear trends could be established as a function of the laminate thickness. In addition, the laminate thickness had little effect on the deflection values. In some cases, it was observed that the deflection was reduced with the increase of the thickness. However, the variation also depended on the effect of the final cross-sections reached for a specific thickness and purlin span.

5.2.3 Construction cost per square meter

Briefly, Table 8 shows all the results obtained through the economic optimisation, expressed as euros per square meter of the roof structure, depending on the span and typology of the trusses, the laminate thicknesses and the roof length.

As it can be observed in Table 8, the main aspects arising from this study are:

(i) the truss types comprised of fewer members resulted in the most economical solutions.

(ii) the smaller laminate thickness, 35 mm, also generated the most economical results. For the most cost-effective scenarios, i.e. trusses of fewer members, and roof length equal to three times the truss span (equivalent to having infinite roof length), an average cost saving around 3% was noticed when the 35 mm laminate thickness was employed. Regarding the 40 and 45 mm thickness, none prevailed over the other as a better alternative, Fig. 12.

Fig. 12. Cost (€ m^{-2}) for the different truss span in the most cost-effective scenarios (i.e. selected truss typology and roof length equal to three times the truss span).

(iii) For the same truss types and laminate thickness, the increase of the length-to-span decreased the cost due to the lower cost influence of the trusses in the entire structure.

5.3. Differences between the individual trusses and the entire roof structure optimisations

Since the 3DGM optimisation was carried out for a variable spacing between trusses, the results obtained could not be directly compared to those arising from the 2DGM optimisation, as different loads were transmitted to the trusses in each model. Nonetheless, it has been noticed that both the 2DGM and 3DGM approaches offered similar results for the 22.5 m span trusses. For both cases, the most cost-effective solution came from the trusses comprised of fewer members and made of the 35 mm thickness. Moreover, the truss cost was similar, except for one case in which the 3DGM optimisation exhibited a lower cost due to a lower truss separation than the 4 m considered in the 2DGM, which resulted lower loads applied on the trusses, as well as the different approach followed in each method to consider the weight of purlins, i.e. in the 3DGM, the weight was automatically introduced by the algorithm as an exact figure according to the specific cross-section, length and spacing for each iteration, whereas a generic load was used in the 2DGM optimisation.

Regarding the dimensions of the elements comprising the trusses, similar height to width ratios were reported for each set of members: upper chords, bottom chord and intermediate members. The specific dimensions were also similar, although with small variations in the cross-section height (± 1 laminate) as well as in the width due to the different truss separation values.

Therefore, although both approaches have been considered valid, the 3DGM optimisation produced better adjustments according to the authors' discretion. Moreover, the 3DGM method provides more data to define the structure, since the algorithm is the one responsible for finding out the optimum parameters, such as truss typology, cross-sections, separation of purlins, separation of trusses, etc.

5.4. Method for pre-dimensioning

This section aims to state the general pre-dimensioning rules for glulam timber roof structures similar to those studied in this work and subjected to loads close to those established paper. Therefore, some guidelines on the general behaviour of the algorithm were drawn.

Optimal results were analysed taking into account a roof length equal to three times the truss span and the different values of truss span and board thickness studied in this research work. The objective was to highlight the most significant or limiting parameters and use them to construct simple equations that could assist an structural engineer with the pre-dimensioning of a structure near to the optimum solution. Nevertheless, it is worth mentioning that the recommended value should always be inferior to the optimum to avoid exceeding it.

Firstly, the analysis of the aforementioned parameters allowed to set a series of rules for the initial dimensioning of any truss span:

- Truss type: $n6$ for trusses up to 22.5 m and $n10$ for trusses over 22.5 m and up to 30 m.

- Regarding the cross-sections, the following considerations should be regarded: The minimum commercial laminate width considered was 90 mm. The result of the pre-dimensioning equations must be rounded to the immediate lower commercial width and the number of laminates (n_{lam}) must be rounded to the nearest whole number. Then, the starting height should be the minimum height required to comply with the dowel spacing; initially, it may be considered two rows of dowels, but the final dowel configuration would depend on the joint calculation. Subsequently, the following rules would apply:

a) Purlins:

- Cross-sections; width (b): 90 mm; height (h): 5 laminates for a 35 mm thickness and 4 for a 40 and 45 mm thickness.

- Length of the purlins (which coincides with truss separation): 4 m for a 40 mm thickness and 4.5 for a 35 and 45 mm thickness.

- Spacing between purlins: 1.25 m

b) Bottom chord, cross-sections:

- Height (h): 4 laminates, in all cases

- Width (b, mm): according to Eq. (3), where L (m) is the truss span:

$$b = 2.666\,L + 46.66 \tag{3}$$

c) Intermediate members, cross-sections:

- Heights (h): 3 laminates for a 45 mm thickness and 4 laminates for a 35 and 40 mm thickness.

- Width (b, mm): according to Eq. (4), where L (m) is the truss span:

$$b = 2.222\,L + 53.33 \quad (4)$$

d) Upper chord, cross-sections:

- Width (b, mm): according to Eq. (5), where L (m) is the truss span and L_r (mm) is the lateral restraining spacing:

$$b = \frac{Lr}{0.426\,L + 11.26} \quad (5)$$

- Height (h): defined by the number of laminates (n_{lam}) according to Eq. (6) and (7):

$$n_{lam} = \frac{r}{0.016\,r + 0.64} \quad (6)$$

Where r is:

$$r = \frac{L}{n} \quad (7)$$

and n is the number of divisions that define the joints in the upper chords (n6, n10 and n14).

Table 9 shows the results obtained by the 3DGM and the pre-dimensioning rules for the structural calculation of an entire roof with a length equal to three times the truss span (which can be considered equivalent for a greater roof length due to the inalterability of the cost per m^2 from that point forward) and different truss span and board thickness.

As observed in Table 9, the number of laminates in the cross section obtained through the pre-dimensioning method coincided with the result proposed by the 3DGM optimisation for most cases. Nevertheless, for some cases in the top chord, the pre-dimensioning method resulted in a cross-section comprised of one less board, i.e. the optimum was not exceeded, which would be corrected in the subsequent calculations of the structure. Regarding the widths of the cross-sections, variations were observed for some cases and the pre-dimensioning method indicated values that differed, on average, by less than 8%. However, an outlier was noticed for the n6 configuration of the 15 m span truss. Only at one point in the upper chord, the width exhibited a greater difference, around 15%, due to a greater variability in the cross-section proposed by each optimisation technique. Nevertheless, the proposed method resulted in a reliable approach to obtain pre-dimensioning results close to the structural optimum, which logically does not exempt the structure to undergo a detailed final calculation to ensure the compliance with the established calculation rules.

6. Conclusions

The GAs have proven to be a valid method to optimise glued laminated timber structures when the laminate thickness is considered. The optimisation was carried out successfully both at the truss level (2D) and the roof structure level (3D). Moreover, it was revealed that the most realistic and appropriate procedure for optimising glulam structures is to take into account the actual thickness and width of the laminates boards of timber.

For the 2D optimisation, the GA obtained the best solutions at an initial population of 150 individuals, an elitism operator of 7%, a crossover probability of 80 % and mutation rate of 1%. Similarly, the best solutions in the 3D model were obtained for an initial population of 330 individuals, an elitism operator of 10%, a crossover probability of 80 % and mutation rate of 1%. The comparison between the 2D discrete optimization and the continuous optimisation reported by Villar et al. (2016) has led to the following conclusions:

- The most economical solutions were also obtained when trusses comprised of fewer members were considered.

- The best cost results were obtained for the smaller laminate thickness, 35 mm.

- For the most economical truss, n6 and 35 mm laminate thickness, the cost increase obtained in the discrete approach was only 1.12% higher compared to the continuous optimisation, which resulted in a mostly similar cross-sections for both models.

- It was observed that the use of the Genetic Algorithms optimisation resulted in lower costs than those obtained through NLP (non-linear programming), (Simon Šilih et al., 2005), even when the laminate thickness was taken into account in the GA model.

Regarding the 3D discrete optimization for an entire roof structure, the results obtained have led to the following conclusions:

- In a spatial roof structure as the one described in this work, the use of trusses comprised of fewer members also rendered the most economic solutions.

- Similarly, the best economical results were obtained for the smaller laminate thickness, 35 mm.

- In terms of truss separation, it was observed that the GA proposed values commonly used in the practice, i.e. 4 m for a 40 mm laminate thickness and 4.5 m for laminate thickness of 35 and 45 mm.

- Regarding the separation between purlins, the algorithm always found the optimum at the maximum allowed separation, which is consistent with the usual execution procedure of roof structures with purlins.

- Depending on the laminate thickness, recommendations for typology, members cross-sections and spacing between trusses and purlins may be proposed in order to obtain optimised glulam roof structures. Therefore, a simplified method of pre-dimensioning has been proposed to obtain a 3D structural arrangement close to the optimum.

- The authors consider the 3D optimisation represents the preferred alternative to set the optimum design and cost adjustment for the roof timber structures.

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Figure 1

Click here to access/download;Figure;Fig 1.tiff

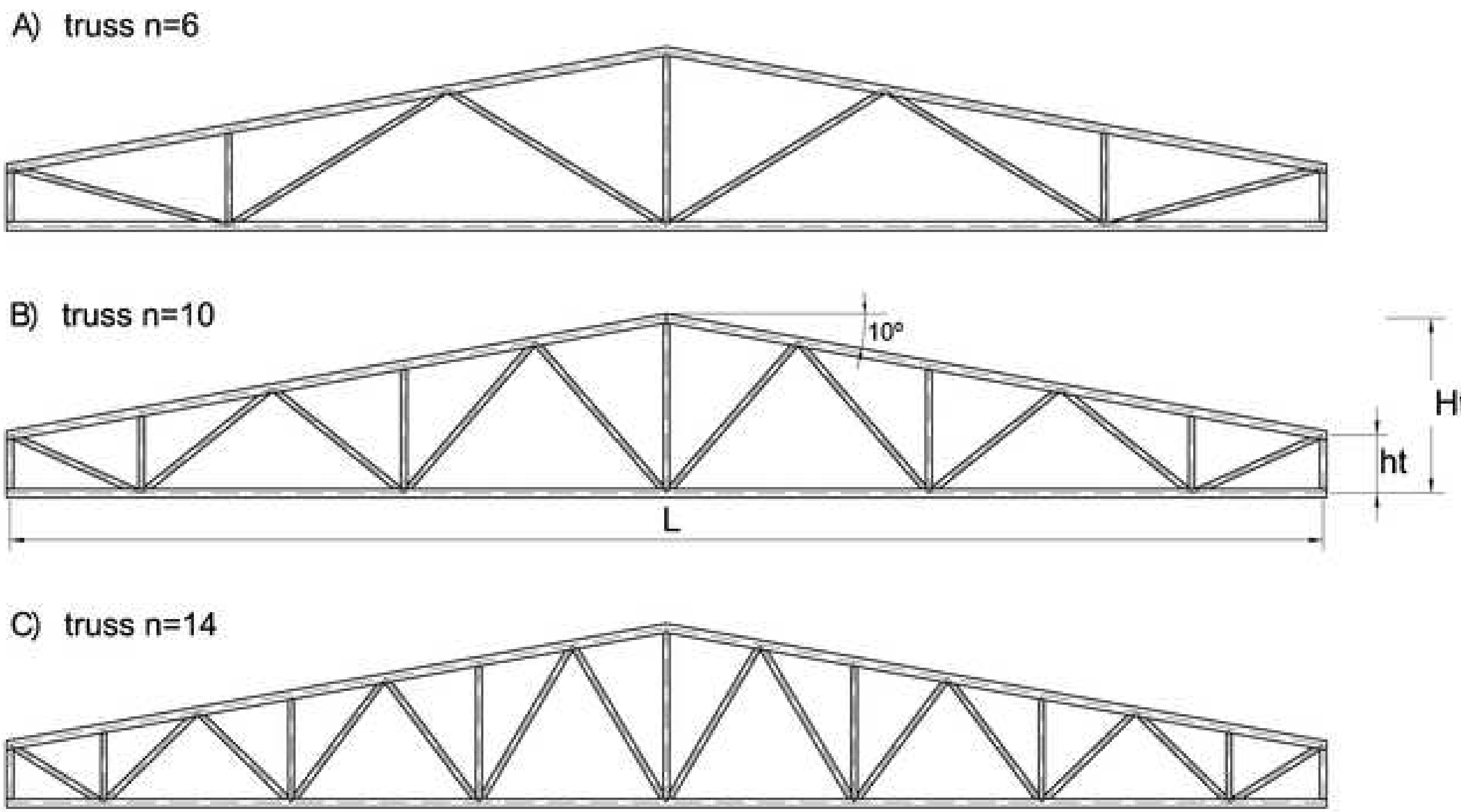


Figure 2

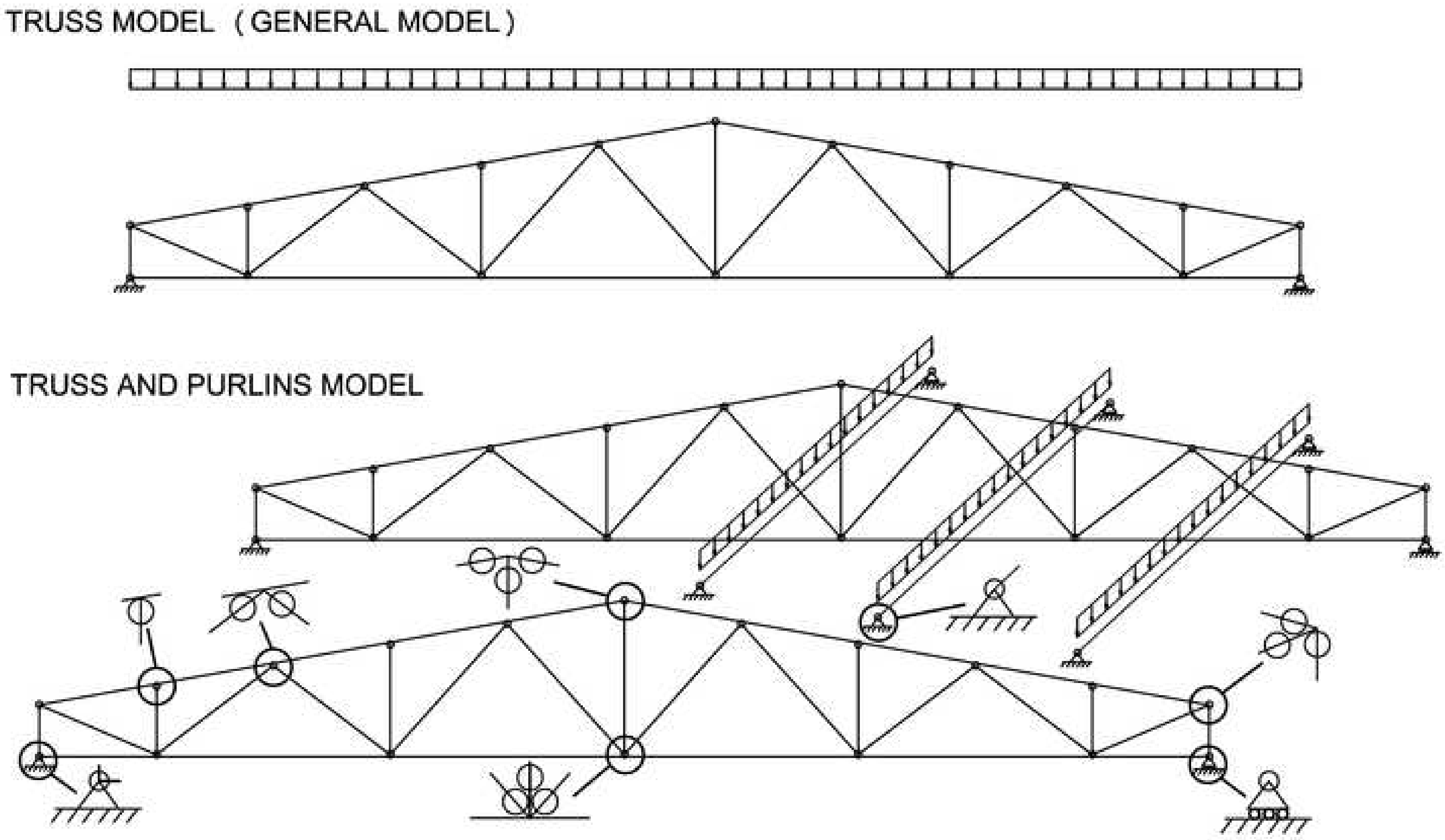


Figure 3

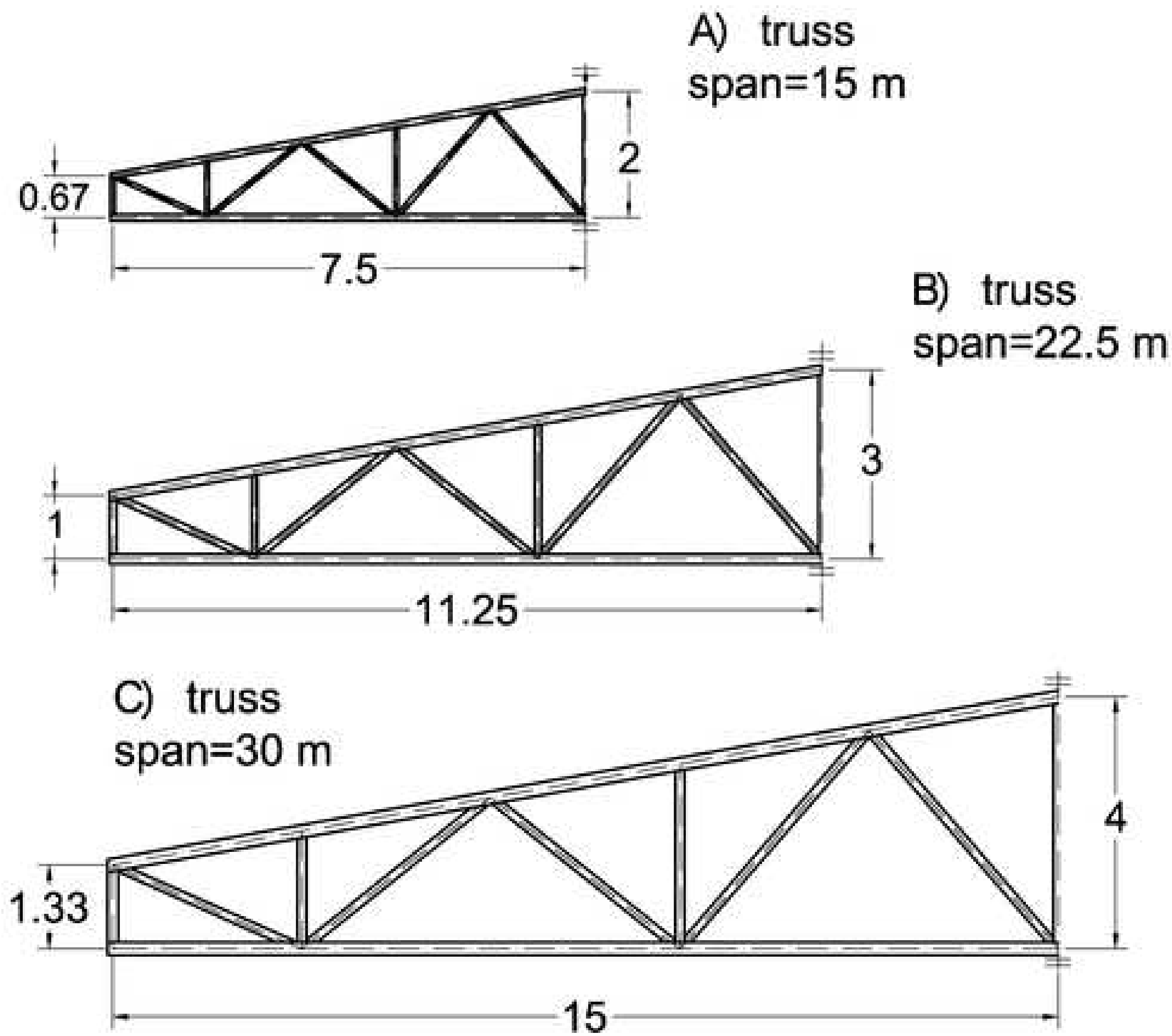

A) truss
span=15 m
0.67
2
7.5
B) truss
span=22.5 m
1
3
11.25
C) truss
span=30 m
1.33
4
15

Figure 4

Click here to access/download;Figure;Fig 4.tiff

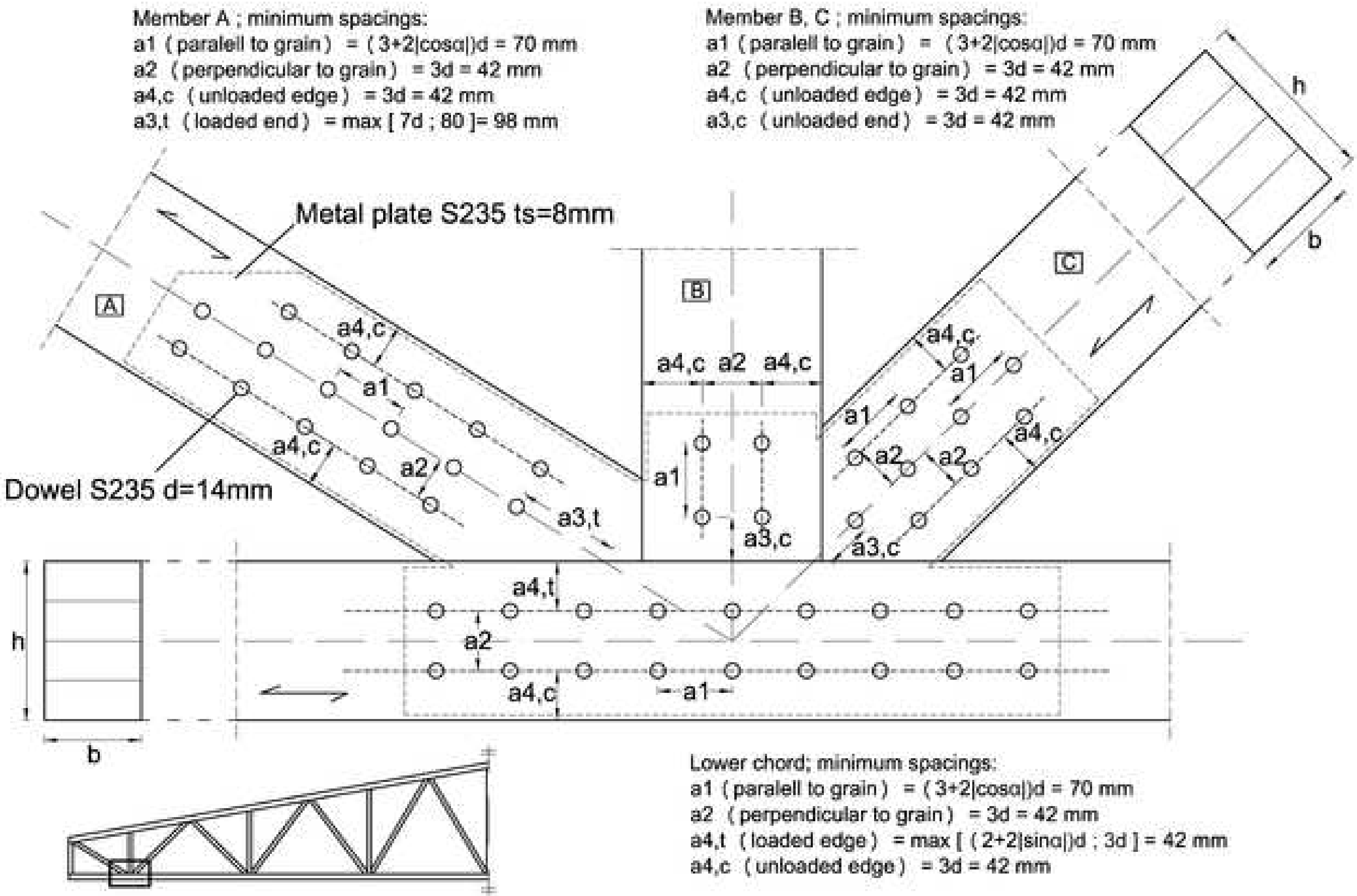


Figure 5

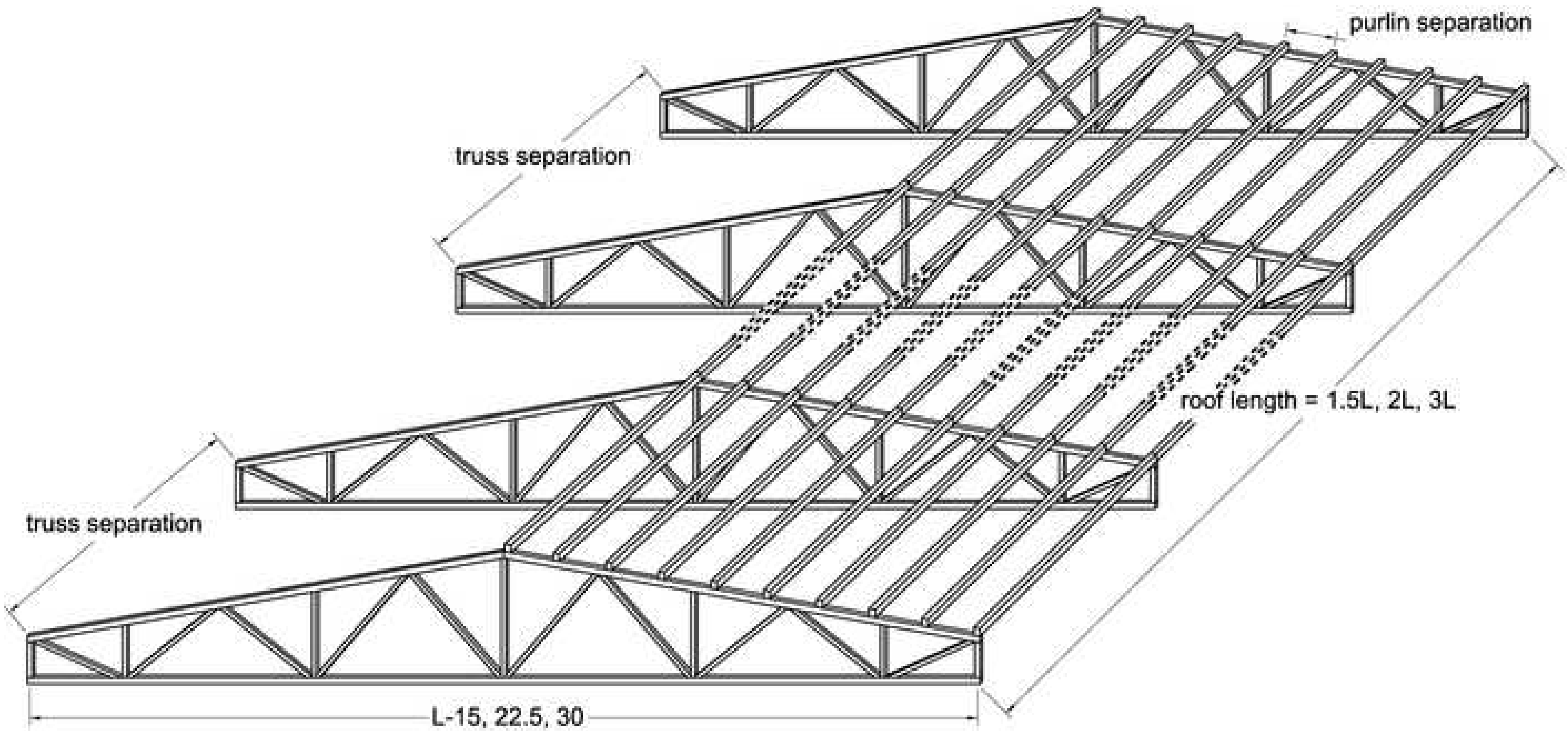

purlin separation
truss separation
truss separation
roof length = 1.5L, 2L, 3L
L-15, 22.5, 30

Figure 6

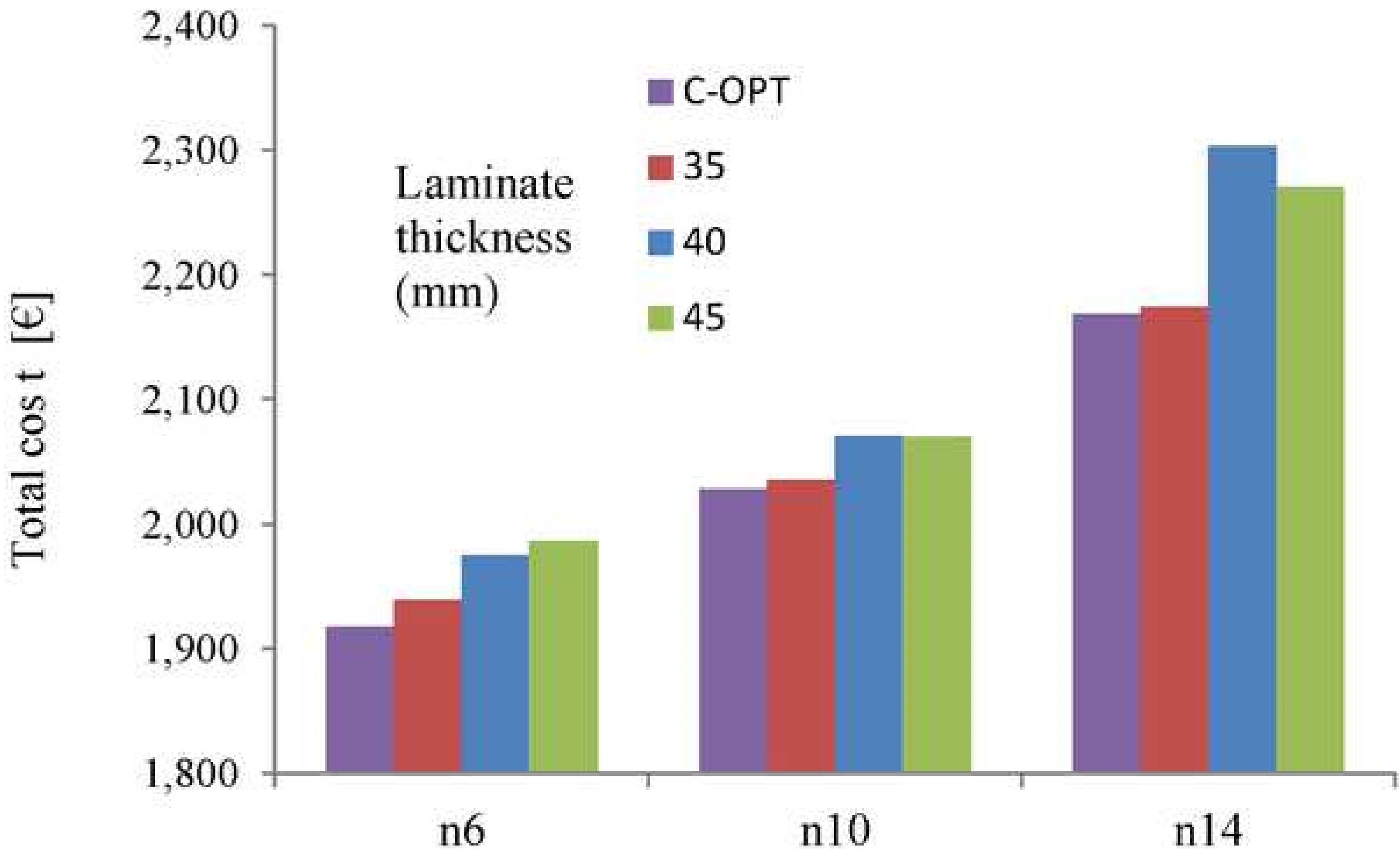

2,400
2,300
2,200
2,100
2,000
1,900
1,800
Total cos t [€]
Laminate thickness (mm)
C-OPT
35
40
45
n6
n10
n14

Figure 7

Click here to access/download;Figure;Fig 7.tiff

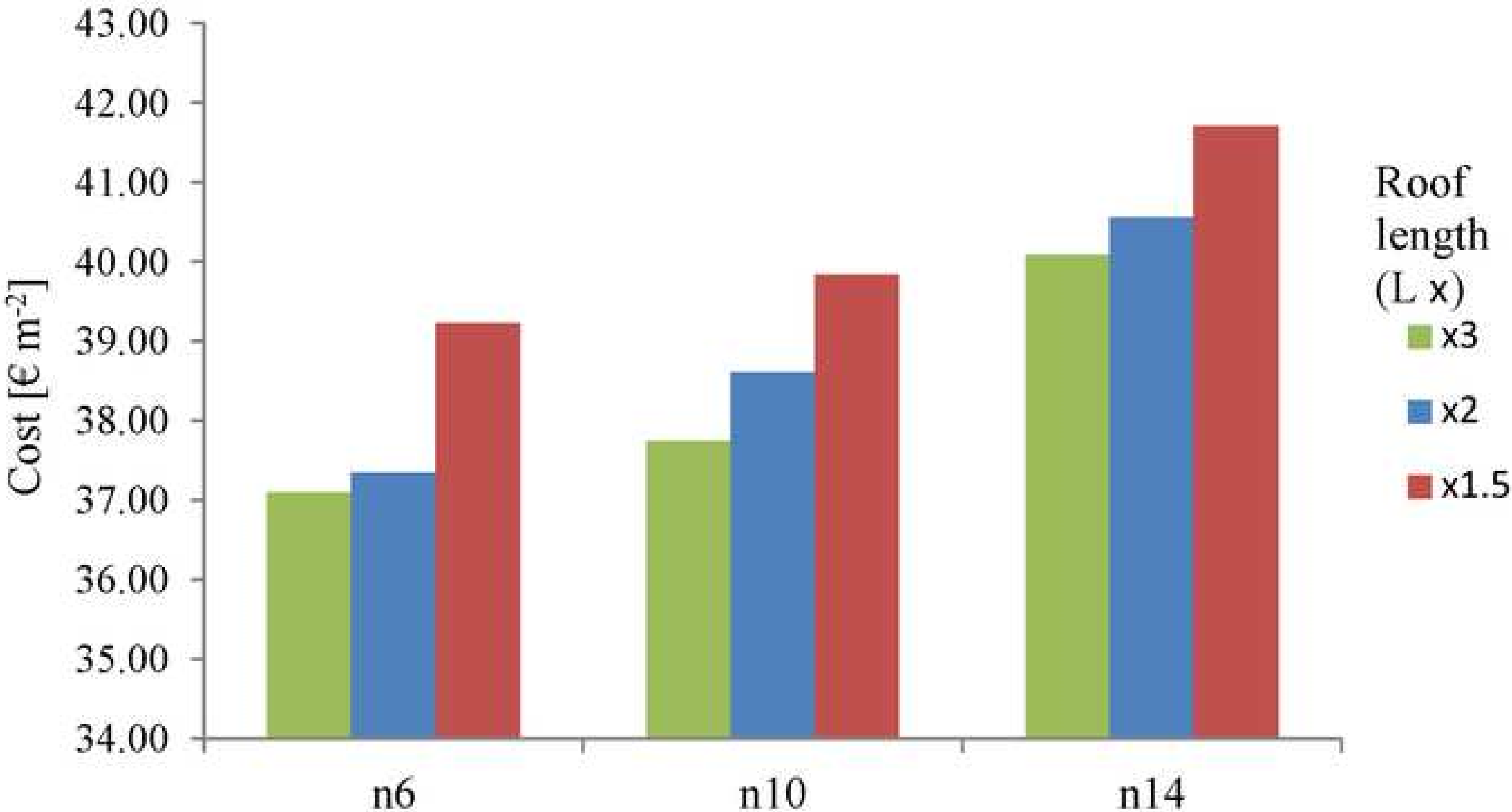
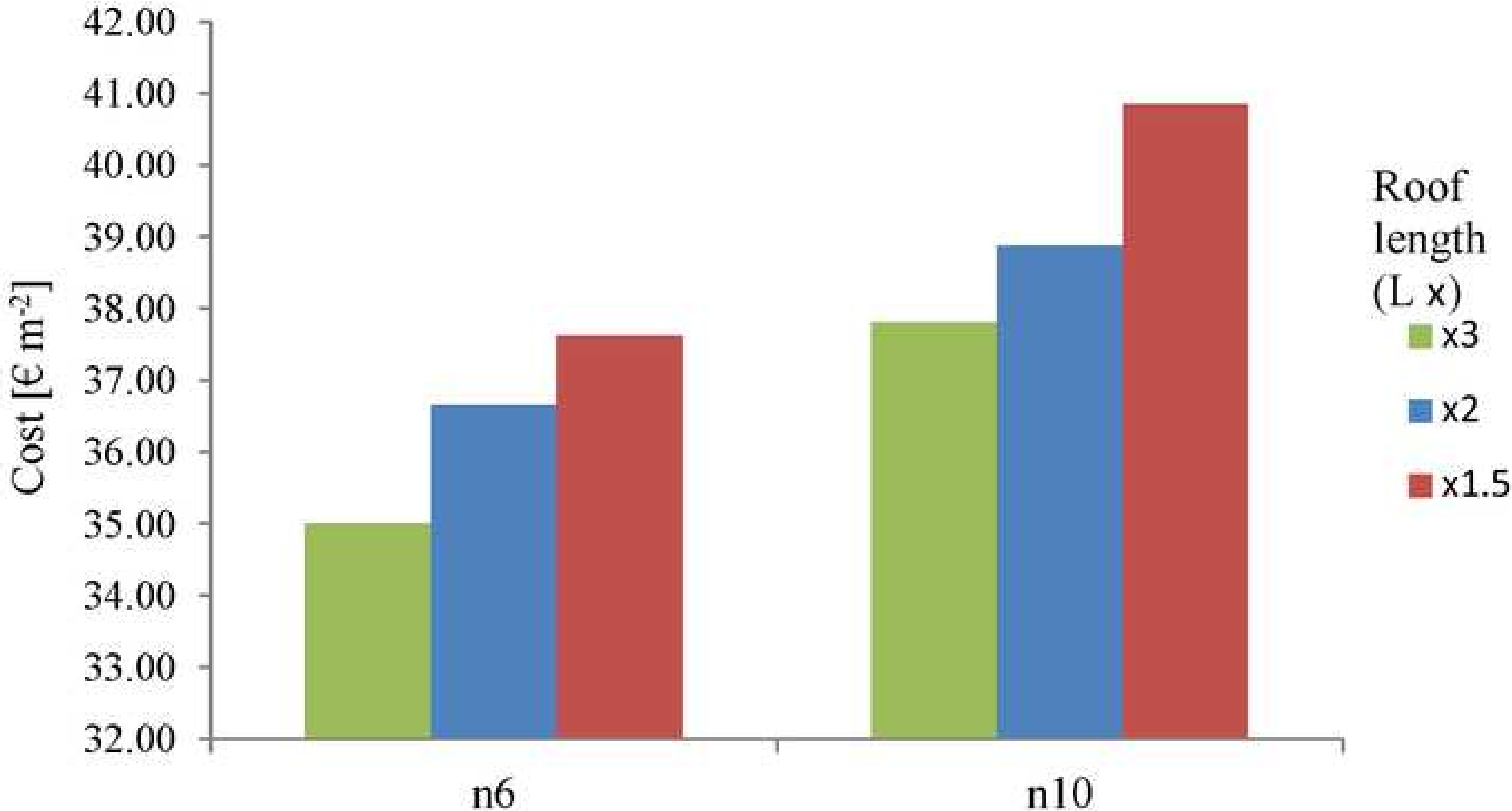


Figure 8



42.00
41.00
40.00
39.00
38.00
37.00
36.00
35.00
34.00
33.00
32.00
Cost [€ m-2]
n6
n10
Roof length (L x)
x3
x2
x1.5

Figure 9

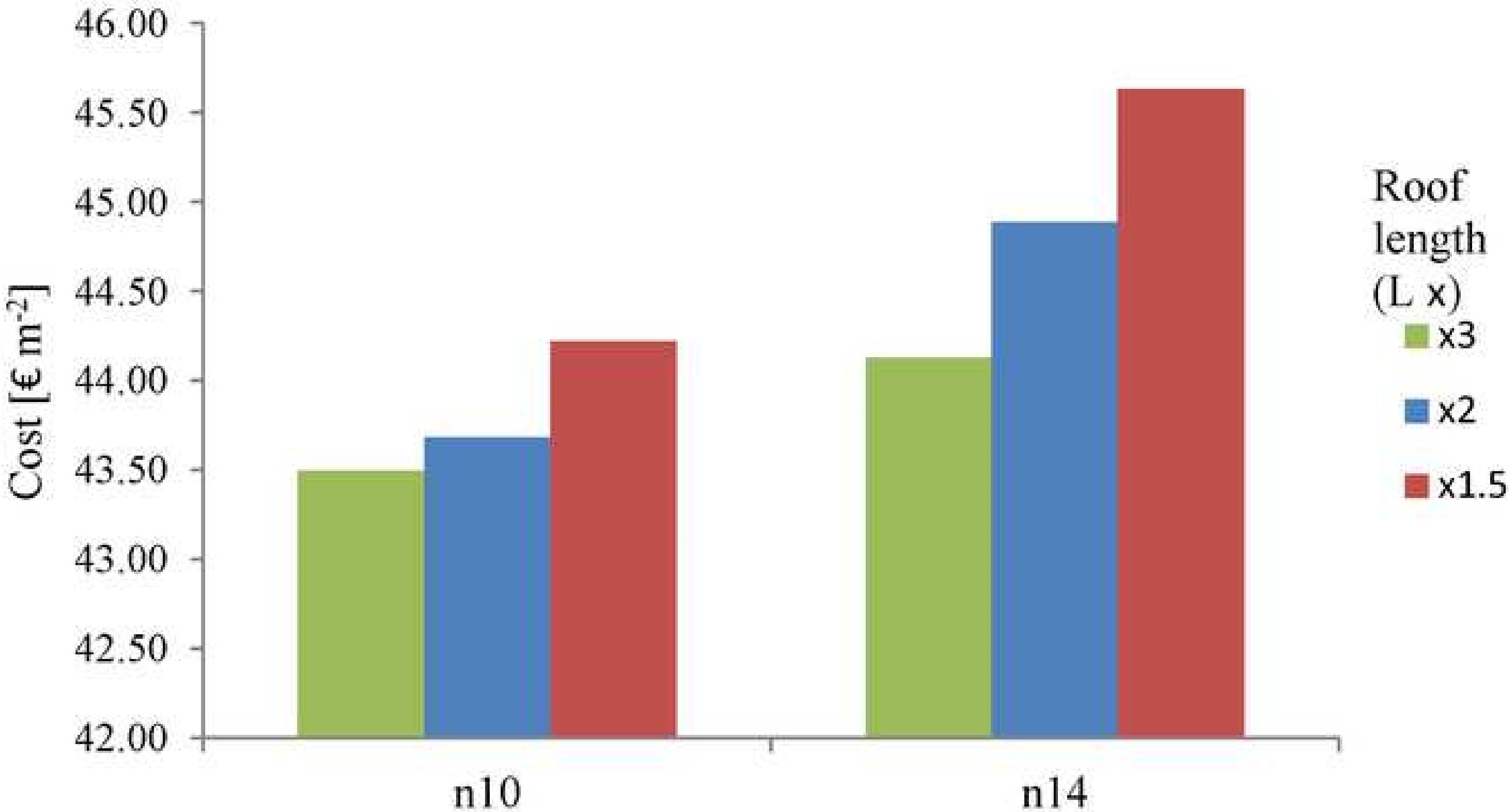
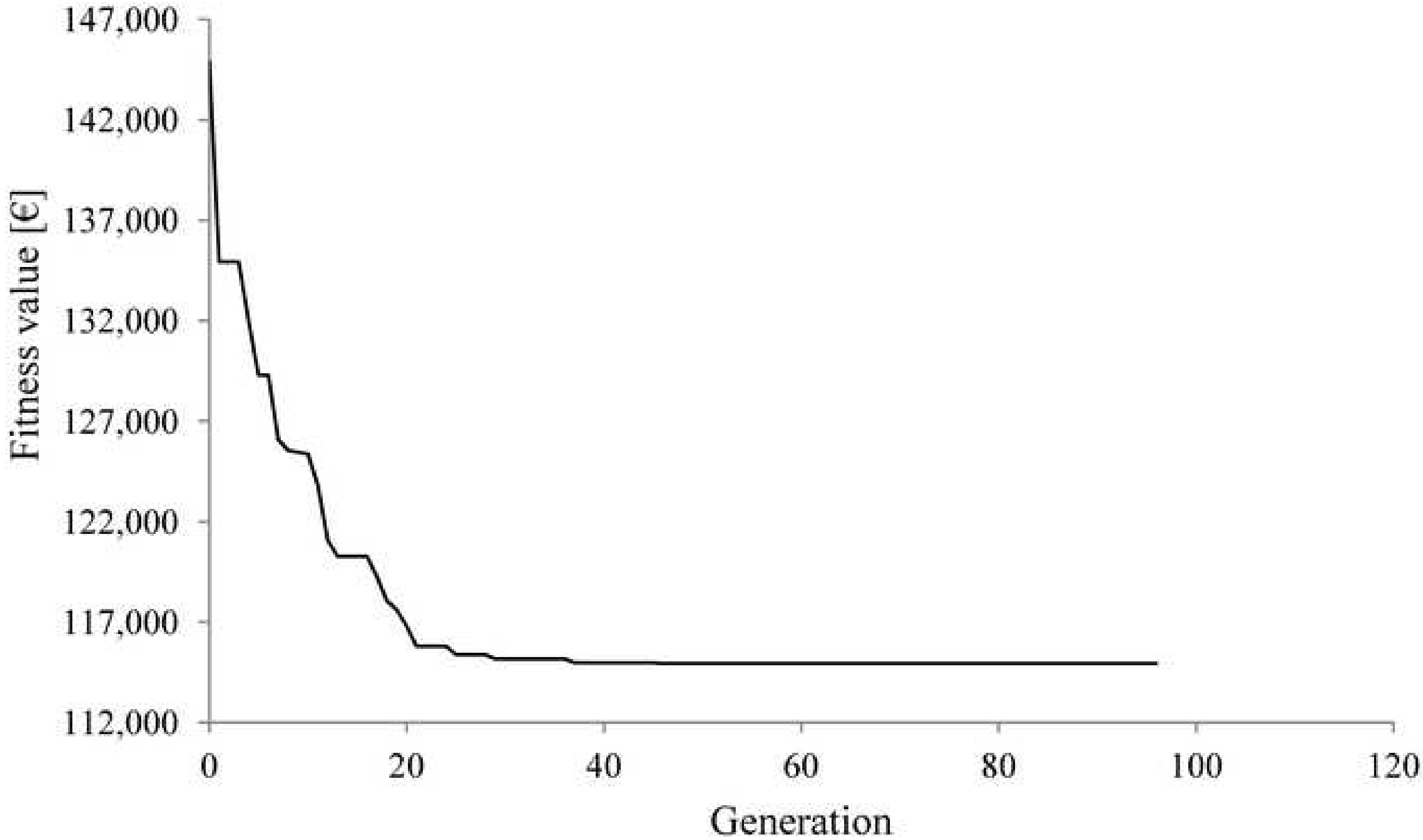
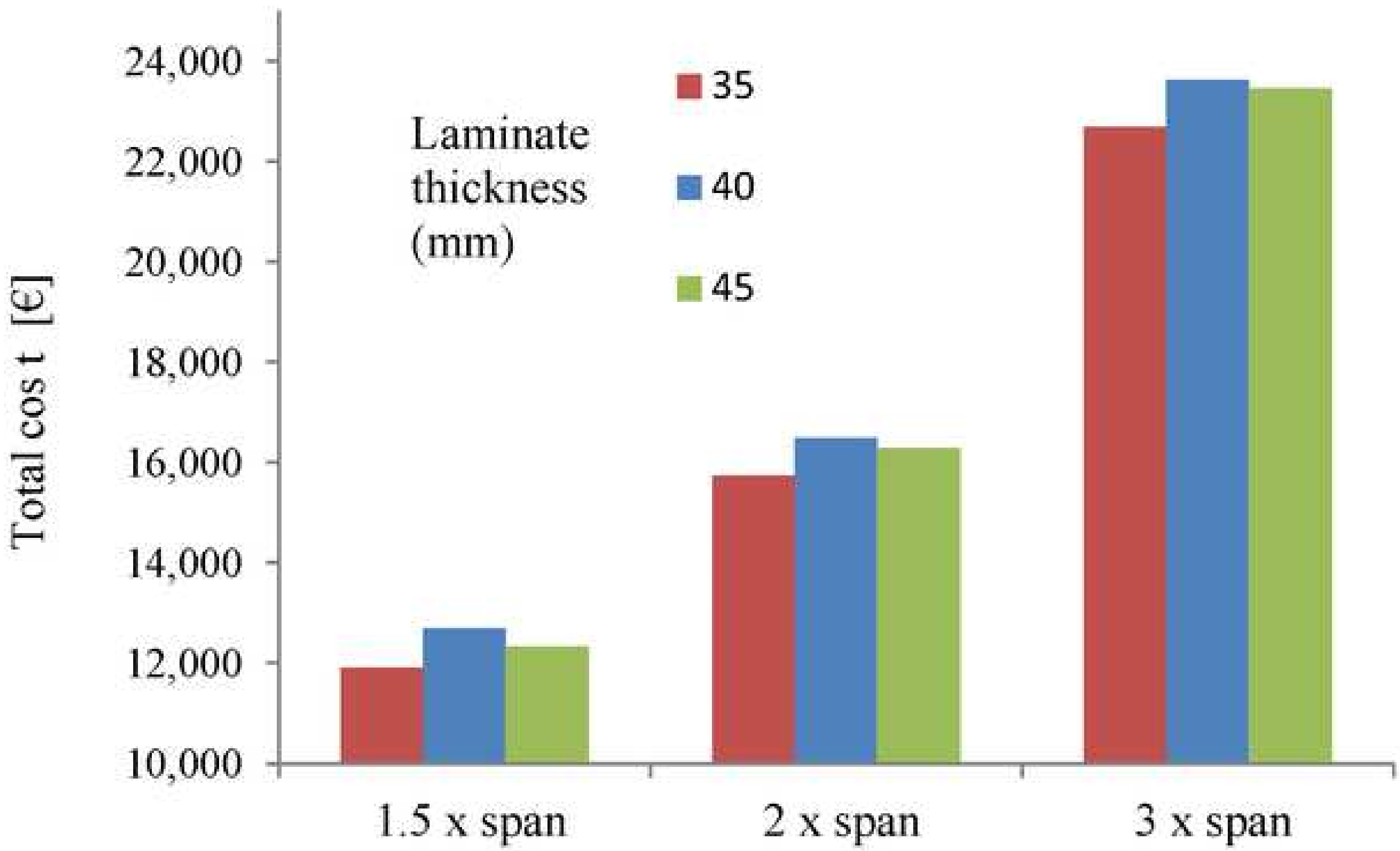

46.00
45.50
45.00
44.50
44.00
43.50
43.00
42.50
42.00
Cost [€ m-2]
n10
n14
Roof length (L x)
x3
x2
x1.5

Figure 10

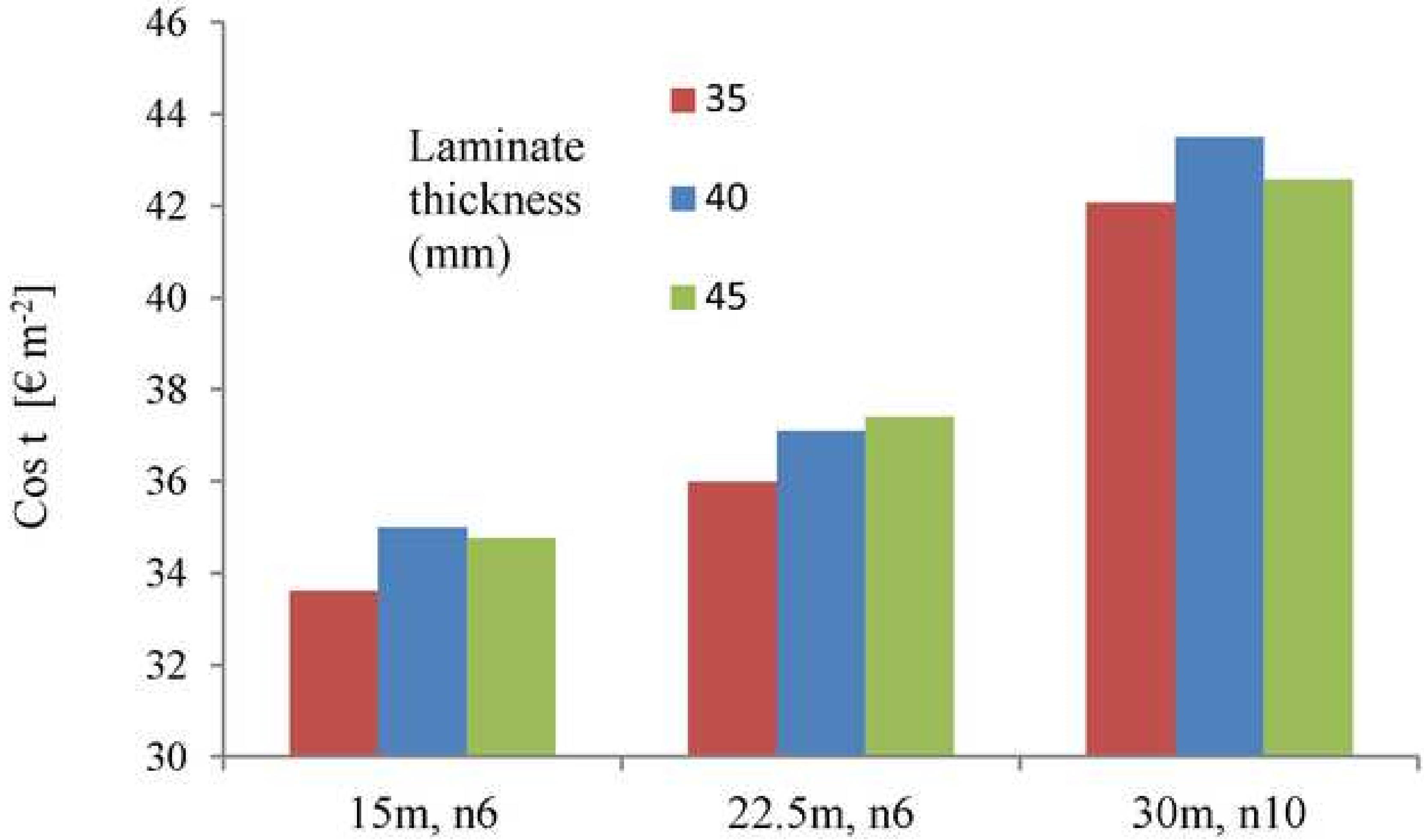


147,000
142,000
137,000
132,000
127,000
122,000
117,000
112,000
Fitness value [€]
0
20
40
60
80
100
120
Generation

Figure 11



Total cos t [€]
24,000
22,000
20,000
18,000
16,000
14,000
12,000
10,000
Laminate thickness (mm)
35
40
45
1.5 x span
2 x span
3 x span


46
44
42
40
38
36
34
32
30
Cost [€ m-2]
Laminate thickness (mm)
35
40
45
15m, n6
22.5m, n6
30m, n10