INTERNATIONAL TRANSACTIONS IN OPERATIONAL RESEARCH

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INTERNATIONAL TRANSACTIONS IN OPERATIONAL RESEARCH

Intl. Trans. in Op. Res. 0 (2021) 1–27
DOI: 10.1111/itor.13013

Assessing Panamanian hospitals' performance with alternative frontier methods

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Received 4 August 2020; received in revised form 14 March 2021; accepted 18 May 2021

Abstract

In the last decades, there has been a growing interest in measuring the efficiency of hospitals using different methodological approaches, mainly represented by data envelopment analysis (DEA) and stochastic frontier analysis (SFA). In this study, we estimate efficiency measures of performance for a sample of Panamanian public hospitals over an 11-year period (2005–2015) using both traditional methods (DEA and SFA) and compare them with efficiencies estimated with an alternative approach, the so-called StoNED (stochastic semi-nonparametric envelopment of data), which combines the virtues of those methods in a unified framework. One of the most interesting features of the public health system in Panama is that it is segmented, as hospitals are operating under two parallel management schemes (the Ministry of Health and the Social Security Fund), thus in our empirical analysis we will also focus on exploring the differences between hospitals operating under each regime. Our results show that there are certain divergences in the efficiency scores estimated with different methodologies, but for all of them it is possible to detect that Panamanian hospitals experienced a clear decrease in their efficiency levels throughout the period evaluated, being this much higher in the hospitals belonging to the Social Security Fund.

Keywords: hospitals; efficiency; healthcare; data envelopment analysis; stochastic frontiers; stoNED

1. Introduction

Improving efficiency levels in the provision of healthcare services has become a priority for health systems worldwide in response to the need to optimize the use of scarce resources in a sector and ensure the provision of public services that better satisfy population needs (Hollingsworth, 2013). This

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is particularly relevant in developing countries where there is a pressing need for proper resource allocation given the limited level of overall infrastructure, resources, and health budget (Hafidz et al., 2018). Within this group are the Latin American and Caribbean (LAC) countries, most of which have significantly increased their public health expenditures in recent decades and will have to continue to do so in the future to serve an increasingly aging population (Moreno-Serra et al., 2019; Akkan et al., 2020). In this context, policymakers and healthcare managers are under increased pressure to exert comprehensive control over resources allocated to healthcare providers (Ferreira et al., 2020).

This research focuses on the study of the health sector in one of those LAC countries, Panama, where the volume of budgetary resources allocated to the health sector has experienced strong growth in recent years (almost doubling in size between 2005 and 2015, whereas population only increased 20% in the same period) because of the increase in staff salaries, the creation of new health facilities and the large increase in budget allocations for medicines and medical and surgical equipment (Ministry of Health, 2015). Moreover, it is important to highlight that the public health Panamanian system has a segmented structure, with two differentiated management models coexisting side by side and covering populations in the same geographic territory: the Social Security Fund and the Ministry of Health. This fragmentation in the financing of healthcare, which is relatively common in LAC countries[1], has negative consequences in terms of the efficiency of the system, as it duplicates administrative costs and makes it very difficult to centralize the mechanisms for strategic purchases from service providers. Therefore, it is not surprising that there is an enormous interest in incorporating structural improvements in the model with the aim of reducing costs and avoiding inefficiencies.

Specifically, we analyze the performance of Panamanian hospitals, as they account for the highest proportion of total healthcare expenditures in the country. Specifically, we examine a sample of 22 hospitals over an 11-year period (2005–2015) so that we can evaluate how their efficiency levels have evolved over the period and whether belonging to one management scheme or the other has influence on their efficiency levels. Moreover, the estimation of efficiency measures should be useful for identifying the most and least efficient hospitals, which may help hospital administrators in benchmarking and establishing systems of rewards and/or penalties.

Generally, efficiency scores of healthcare institutions can be estimated using either parametric or nonparametric methods. Since the pioneer work by Sherman (1984), data envelopment analysis (DEA) has been the most widely applied method to measure hospital efficiency (for extensive reviews of this literature, see O'Neill et al., 2008; Cantor and Poh, 2018; Kohl et al., 2019). This technique provides a measure of the relative efficiency of organizational units according to their use of their resources and the outputs achieved without imposing any functional form on technology, thus it is more flexible and can be easily adapted to the characteristics of healthcare provision. The main shortcoming of this method comes from their deterministic nature, that is, it attributes all deviations from the frontier to inefficiency without consideration for measurement errors, sample noise, or specification errors. As an alternative, parametric approaches assume a specific functional form of the boundary of the production set with constant parameters to be estimated. In this case, deviations from the efficiency frontier (measured by the error term) can be decomposed into two

[1] In many countries, there is usually one social health insurance scheme mostly for the formal sector and a national health system that guarantees coverage for the poor and those in the informal labor market (Medici and Lewis, 2019).

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parts, one representing randomness (or statistical noise) and the other inefficiency, by applying stochastic frontier methods (SFA). Their main disadvantage is that they require specifying a functional form for the production frontier and relying on strong distributional assumptions about the inefficiency term.

Those approaches are frequently identified as competitors, thus in the literature we can find a large number of studies that have measured hospitals' performance using different approaches and comparing the indicators estimated with those alternatives (e.g., Banker et al., 1986; Linna, 1998; Chirikos and Sear, 2000; Jacobs, 2001; Siciliani, 2006; Katharakis et al., 2014). However, DEA and SFA should not be viewed as direct opponents, but rather complements, as the main strength of each method (consideration of noise in SFA and flexibility in DEA) is precisely the main limitation of the other method (DEA is deterministic and SFA needs the assumption of a functional form) (Kuosmanen et al., 2015). In fact, there is a growing body of literature focused on bridging the gap between parametric and nonparametric approaches by combining the virtues of both alternatives to develop semi-parametric methods or nonparametric stochastic frontier methods (Fan et al., 1996; Kumbhakar et al., 2007; Simar and Zelenyuk, 2011)[2].

In the present study, we apply one of those extensions that allows for a full integration of DEA and SFA into a unified framework. Specifically, we rely on the so-called Stochastic semi-Nonparametric Envelopment of Data (StoNED) method developed by Kuosmanen and Kortelainen (2012) by refining the main ideas introduced by Banker and Maindiratta (1992). This method can be interpreted as a generalization of both DEA and SFA, incorporating a classical model of noise into DEA and imposing few microeconomic properties (monotonicity, concavity, and continuity) on the shape of the function estimated in SFA. Therefore, it combines the main virtue of DEA, as the production frontier is estimated without specification of a functional form, with the main advantage of SFA, because it allows for disentangling noise and efficiency. Moreover, this alternative methodology seems to provide more accurate measures of units' performance than traditional methods, especially in noisy scenarios, as demonstrated in several studies conducted within the controlled environment of Monte Carlo simulations (Kuosmanen et al., 2013; Andor and Hesse, 2014). As a result, this method has started to be applied to measure efficiency in empirical studies conducted in different sectors such as electricity distribution (e.g. Kuosmanen, 2012; Saastamoinen and Kuosmanen, 2016), banking (Eskelinen and Kuosmanen, 2013), agriculture (Vidoli and Ferrara, 2015), tax administration (Nguyen et al., 2020), municipal services (Cordero et al., 2020), regional analysis (Polo et al., 2021) or subjective wellbeing (Cordero et al., 2021). However, as far as we know, this methodological approach has not yet been applied to assess the performance of hospitals.

This study contributes to the existing research field focused on assessing hospital performance in several directions. First, we apply a technique (StoNED) that attempts to combine the advantages of DEA and SFA models to estimate efficiency measures for a set of hospitals for the first time. This approach allows us to consider that deviations from the frontier of inefficient Panamanian hospitals might be caused by some unobserved factors (included as part of statistical noise). Likewise, we investigate whether the application of different methods to estimate efficiency measures produce consistent rankings of units by comparing the estimates obtained by applying this method with

[2]Olesen and Petersen (2016) provide an exhaustive literature review of the main alternatives related to stochastic DEA that allow for an estimation of stochastic inefficiency.

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those derived with the traditional DEA and SFA approaches. Second, we focus our attention on assessing the performance of public hospitals operating in Panama, a Latin American country for which no previous empirical study on the hospital sector has been conducted so far. This lack of prior evidence is mainly due to the lack of official and accessible databases on the functioning of the Panamanian hospital system. Unfortunately, this limitation is frequently present in low and middle income countries that frequently lack sufficient data to perform a meaningful efficiency analysis (Au et al., 2014). Therefore, an added value of the present work has been precisely the development of an analytical tool to collect information on a large number of indicators and for an extended period of 11 years (2005–2015) through questionnaires specifically designed for the development of this research. Third, the existence of two differentiated management models coexisting within the same health system (MoH and SSF) allows us to analyze whether there are differences in the levels of efficiency demonstrated by hospitals belonging to each regime and explore how they evolved over the evaluated period.

The remainder of this paper is structured as follows. In Section 2, we briefly review the related literature on efficiency measurement in the hospital sector. In Section 3, we describe the three methodologies applied in this study, that is, DEA, SFA, and StoNED. Then, in Section 4, we explain the main characteristics of the database and the variables used in our empirical study. The main results are presented and discussed in Section 5. Finally, Section 6 summarizes the main conclusions.

2. Literature review

There is a large literature on measuring the productivity and efficiency of healthcare institutions, especially for hospitals, using both parametric and nonparametric approaches (Worthington, 2004). Among all these applications, we can find studies with very different purposes, ranging from studies focused on exploring the effects of different types of ownership (Herr, 2008; Czypionka et al., 2014; Ferreira and Marques, 2021), analyzing the relationship between efficiency and quality (Gok and Sezen, 2013; Wu et al., 2013; Khushalani and Ozcan, 2017; Ferreira and Marques, 2019) or examining the relationship between specialization and efficiency (Langabeer and Ozcan, 2009; Lindlbauer and Schreyögg, 2014).

Most of these studies rely on nonparametric methods and, more specifically on DEA, to calculate efficiency scores as well as other closely instruments such as distance functions or Malmquist indices when panel data are available (Jacobs et al., 2006; Hollingsworth, 2008; Kohl et al., 2019). The main advantages of those methods are that they can easily handle multiple outputs in the transformation process, they provide detailed information on areas of inefficiency and, more importantly, they do not require to specify any functional form that links inputs to outputs.

However, these methodologies have the important limitation that they do not provide any information of the uncertainty due to sample variation, thus it is not possible to perform a statistical sensitivity analysis or applying statistical testing procedures. As a potential remedy to this problem, Simar and Wilson (1998, 1999, 2000) propose several bootstrapping procedures to accurately estimate efficiency and productivity scores and confidence intervals that establish the desired statistical inferences. As these options have become available, their use has become increasingly common for

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statistical testing of estimators calculated with DEA and Malmquist indices in the assessment of hospitals (e.g., Staat, 2006; Chowdhury et al., 2011; Marques and Carvalho, 2013; Ferreira and Marques, 2015; Cheng et al., 2016).

Another potential drawback of nonparametric methods is that they are very sensitive to the presence of extreme points and outliers. A possible solution to this problem consists of using robust partial frontiers (order-m and order-α) proposed by Cazals et al. (2002) and Aragon et al. (2005), respectively[3]. These methods are characterized by the fact that the benchmark for each evaluated unit is not the full frontier formed by the best practices of all units, but a partial frontier that envelop only a reduced number of observations randomly drawn from the sample. As they do not include all the observations, it mitigates the problem caused by the presence of outliers, extreme values, or noise in the data. Given this advantage, it is not surprising that the use of these techniques has recently become popular to assess the efficiency of hospitals (Varabyova et al., 2017; Ferreira et al., 2018; Mastromarco et al., 2019; Ferreira and Marques, 2020).

The use of stochastic frontier models is another possibility to mitigate the two problems mentioned above. This approach offers rich specification of the production process, allows for statistical testing of hypotheses about the production frontier and constructing confidence intervals around the estimated efficiency measures. Moreover, they also perform well with panel data because they take into account potential unobserved heterogeneity due to the use of econometric techniques. All these advantages have led to its widespread use for analyzing hospital performance (e.g., Rosko, 2001; Herr, 2008; Rosko and Mutter, 2008; Barros et al., 2013). However, SFA has two major limitations. The first is the requirement of a large sample size for the applied econometric technique to work well and the second is the reliance on an assumed distribution of efficiency estimates.

As mentioned in the introduction, an obvious alternative would be the combination of both methods using some of the alternatives that have been developed in the literature for this purpose such as the stochastic data envelopment analysis. However, the use of these techniques to measure hospital efficiency is still in its infancy, with few applied studies having applied them to date (Kheirollahi et al., 2015; Mitropoulos et al., 2015).

The vast majority of the aforementioned studies and, in general, most part of the existing literature on the measuring the performance of healthcare providers are referred to developed countries, mainly from Europe and North America (O'Neill et al., 2008). Nevertheless, in the last two decades, there has been a certain growth in the number of studies applied in low and middle-income countries (e.g., Pham, 2011; Şahin and İlgün, 2019; Babalola and Moodley, 2020; Seddighi et al., 2020). Despite this, the available empirical evidence on LAC countries is still very scarce. These include the empirical studies conducted by Arocena and García-Prado (2007) for hospitals operating in Costa Rica, de Castro Lobo et al. (2010) and Longaray et al. (2018) for Brazilian hospitals, Giménez et al. (2019a, 2019b) for Mexican and Colombian hospitals, respectively, or Piedra-Peña (2020) for Ecuadorian hospitals. Therefore, the present paper focused on assessing the performance of Panamanian public hospitals constitutes an important contribution within this area of research.

[3]For a detailed explanation of these approaches, see Daraio and Simar (2007) or Daouia and Gijbels (2011).

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3. Methodology

In the next lines, we introduce the basic notation of the three methodological approaches used in this study considering that the production units use a set of inputs X ($x \in \Re_+^p$) to produce a single output Y ($y \in \Re_+^q$). We assume that all units share the same production technology Ψ where the frontier is given by the maximum output that can be produced given their inputs. Then, the feasible combinations of inputs and outputs can be defined as the marginal attainable set:

$$\Psi = \left\{(x, y) \in \Re_+^{p+q} | x \text{ can produce } y\right\}. \tag{1}$$

Given that the production set Ψ cannot be observed, it has to be estimated from a random sample of production units. Following the seminal work of Farrell (1957), technical efficiency can be interpreted as producing the maximum amount of output from a given amount of input (output orientation) or producing a given output with minimum quantities of inputs (input orientation)[4]. The efficient units (hospitals in our case) will be part of the production frontier and the inefficiency for those who do not belong to the frontier can be defined as

$$\theta(x, y) = \inf\{\theta | (\theta x, y) \in \Psi\}. \tag{2}$$

3.1. *Data envelopment analysis*

DEA is a linear programming technique introduced by Charnes et al. (1978) that relies on the convexity assumption of Ψ. The aim of this method is to build an envelope that includes all the efficient units, together with their linear combinations, leaving the rest of the (inefficient) units below it. Following the notation provided by Daraio and Simar (2007), this estimator $\hat{\Psi}_{DEA}$ can be defined as

$$\hat{\Psi}_{DEA} = \left\{(x, y) \in \Re_+^{p+q} \mid y \leq \sum_{i=1}^{n} \gamma_i \cdot y_i;\ x \geq \sum_{i=1}^{n} \gamma_i \cdot x_i,\ \text{for } (\gamma_1, \ldots, \gamma_n) \right.$$
$$\left. \text{s.t.} \sum_{i=1}^{n} \gamma_i = 1; \gamma_i \geq 0, i = 1, \ldots, n \right\}. \tag{3}$$

This model implicitly assumes variable returns of scale in production according to the model introduced by Banker et al. (1984). Thus, inefficient units are only compared with others that operate on the same scale. In this way, the technique is made more flexible by facilitating the analysis in those cases (very common) in which not all the units evaluated operate on a similar scale. The

[4]In this paper, we adopt an input orientation because hospital managers generally have more control over their inputs than their outputs. Moreover, in most countries the emphasis is on controlling costs rather than increasing demand for healthcare (O'Neill et al., 2008).

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input-oriented efficiency scores for a given observation (x, y) can be obtained by solving a simple linear program:

$$\hat{\theta}_{DEA}(x, y) = \inf\left\{\theta | (\theta x, y) \in \hat{\Psi}_{DEA}\right\}, \tag{4}$$

where $\hat{\theta}_{DEA}$ denotes an efficient unit, while $\hat{\theta}_{DEA} < 1$ implies that the hospital is inefficient. DEA typically relies on cross-sectional data to estimate performance, which makes it difficult to obtain a view of the evolution of efficiency over time. Therefore, when longitudinal data are available, DEA needs to be adapted to a dynamic framework.

In the literature, we can find different alternatives to measure efficiency with panel data (Linna, 1998), but the most widely used approach for this purpose is DEA window analysis, which is based on the principle of moving averages (Charnes et al., 1984). This approach suggests pooling all cross-sections together forming an intertemporal production set that uses and treats separately all observations from all time periods. The basic idea within this framework is to regard each unit, in this case each hospital, as if it were a different unit in each of the reporting periods. Thus, the performance of a unit in a particular period is compared with its own performance in other periods as well as with the performance of other units (Harrison et al., 2009; Kazley and Ozcan, 2009). This procedure has the advantage of increasing the number of observations available in the analysis, which can be useful when the sample size is small as in our empirical application. Nevertheless, we should bear in mind that when using this method we are implicitly assuming that there are no substantial technical changes over the entire time period, as all units within a given window are measured against each other.

3.2. Stochastic frontier analysis

Contrary to the DEA explained above, this approach is based on an econometric (i.e., parametric) specification of the functional form of the production frontier with constant parameters (β). The main exponent is the SFA model introduced by Aigner et al. (1977) and Meeusen and van Den Broeck (1977), which estimates the unknown parameters (β) from the actual observations using maximum likelihood techniques. Using a Cobb–Douglas production function, this method can be expressed as follows:

$$\ln y_i = \beta_0 + \sum_{j=1}^{p} \beta_j \cdot \ln x_{i,j} + \epsilon_i, \tag{5}$$

where the error term (ϵ) is composed of two independent components, $\epsilon_i = v_i - u_i$ $(i = 1, \ldots, n)$. The first element, v_i, is a random variable reflecting noise and other stochastic disturbances like a temporary local outbreak of disease or some unexpected expenditures for repairs as well as potential measurement errors resulting from the nonconsideration of relevant variables in the model. This term is assumed to be an independent and identically distributed normal random variable with zero mean and constant variance. The second component, u_i, captures technical inefficiency

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(distance from the frontier) and it is assumed to follow a one side distribution (e.g., half-normal, truncated normal, exponential), as it can only affect output in one direction.

We assume here a half-normal distribution for the inefficiency term ($u_i \rightarrow |N(0; \sigma_u^2)|$) and a normal distribution for the noise term ($v_i \rightarrow N(0; \sigma_v^2)$). Under these assumptions, the marginal density function of the composed error term is defined by

$$f(\epsilon) = \frac{2}{\sigma}\phi\left(\frac{\epsilon}{\sigma}\right)\Phi\left(-\frac{\epsilon\lambda}{\sigma}\right), \tag{6}$$

knowing that $\sigma = \sqrt{\sigma_u^2 + \sigma_v^2}$, $\lambda = \frac{\sigma_u}{\sigma_v}$ and ϕ and Φ are the standard normal cumulative distribution and the density function, respectively. To estimate σ_u, σ_v, and ϵ_i, we use the maximum likelihood procedure. Thus, the corresponding likelihood function of Equation (6) that must be maximized is represented by

$$L(\alpha, \beta, \sigma, \lambda) = C - n \cdot \ln\sigma + \sum_{i=1}^{n} \ln\Phi\left(-\frac{\epsilon\lambda}{\sigma}\right) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}\epsilon_i^2, \tag{7}$$

where $\epsilon_i = y_i - (\beta_0 + \sum_{j=1}^{p} \beta_j \cdot \ln x_{i,j})$.

Once the parametric functional form has been estimated, it is also possible to derive individual efficiencies. To do this, we use the consistent point-estimator proposed by Battese and Coelli (1988):

$$\hat{E}(\exp(-u_i)|\hat{\epsilon}_i) = \frac{\phi\left(\frac{\hat{\mu}_{*i}}{\hat{\sigma}_{*u} - \hat{\sigma}_{*v}}\right)}{\phi\left(\frac{\hat{\mu}_{*i}}{\hat{\sigma}_{*u}}\right)} \cdot \exp\left(\frac{1}{2}\hat{\sigma}_*^2 - \hat{\mu}_{*i}\right), \tag{8}$$

where $\hat{\mu}_* = -\frac{\hat{\epsilon}_i\hat{\sigma}_u^2}{\hat{\sigma}^2}$ and $\hat{\sigma}_*^2 = \frac{\hat{\sigma}_v^2\hat{\sigma}_u^2}{\hat{\sigma}^2}$.

Finally, this model can also be adapted to a dynamic context when longitudinal data are available, that is, when different observations are available for the same unit in different time periods ($t = 1, \ldots, T$), by assuming that the production function is time invariant and common to all units. To estimate this model, it is common to use the fixed effects approach suggested by Schmidt and Sickles (1984), in which usually the inefficiency term (u_i) is assumed not to change over time, but the disturbance term (v_{it}) can. Likewise, we assume that u_i and v_{it} are independent of inputs x_{it} and of each other.

3.3. Stochastic semi-nonparametric envelopment of data

This model is based on the regression interpretation of DEA proposed by Kuosmanen and Johnson (2010) to combine the key characteristics of DEA and SFA into a unified model, that is, the DEA-type nonparametric frontier with the probabilistic treatment of efficiency and noise in stochastic models. In this scenario, the production function is defined in such a way that the deviations of

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outputs from the production frontier can be decomposed into two different sources: the inefficiency term (u_i) and the stochastic noise term (v_i), thus the production function can be defined as

$$y_i = f(x_i) + \epsilon_i = f(x_i) - u_i + v_i \qquad i = 1, \ldots, n, \tag{9}$$

where $\epsilon_i = v_i - u_i$ is the composed error term and it is assumed that variables v_i and u_i are random variables that are statistically independent of each other as well as of inputs. The inefficiency term has a positive mean denoted by $E(u_i) > 0$ and a constant finite variance denoted by $\mathrm{Var}(u_i) = \sigma_u^2 < \infty$. For the noise term, a mean value of zero $E(v_i) = 0$ and a constant finite variance $\mathrm{Var}(v_i) = \sigma_v^2 < \infty$ is also assumed.

We can adapt this technique to a dynamic context if panel data are available (i.e., each unit in $t = 1, \ldots, T$). Again, we follow Schmidt and Sickles (1984) taking the most efficient unit in the sample as the reference, and estimate the time-invariant inefficiency terms u_i (for a detailed description of that procedure, see Kuosmanen and Kortelainen, 2012). Therefore, we can define a time invariant frontier model as

$$\begin{aligned} y_{it} &= \Phi(x_{it}) \cdot \exp(\epsilon_{it}) \\ \epsilon_{it} &= v_{it} - u_i \end{aligned} \tag{10}$$

where y_{it} is the observed output of firm i in time period t, x_{it} is a vector of inputs consumed by firm i in time period t, and Φ is a production function that is time invariant and common to all units.

The estimation of the StoNED model is conducted in two stages. The first stage estimates an average production function with concave nonparametric least squares (CNLS) (Hildreth, 1954). The main advantage of this method is that, like nonparametric methods, it does not assume any *a priori* assumption on a functional form, but establishes only concavity and monotonicity of the production function. Specifically, the estimator can be solved as the optimal solution to the following quadratic programming problem[5]:

$$\begin{aligned} &\min \sum_{t=1}^{T} \sum_{i=1}^{n} \epsilon_{it}^2 \qquad \text{s.t.} \\ &\quad y_{it} = \alpha_{it} + \boldsymbol{\beta}'_{it}\boldsymbol{x}_{it} + \epsilon_{it} \qquad \forall\, i = 1, \ldots, n \quad \forall\, t = 1, \ldots, T \\ &\quad \alpha_{it} + \boldsymbol{\beta}'_{it}\boldsymbol{x}_{it} \leq \alpha_{it} + \boldsymbol{\beta}'_{it}\boldsymbol{x}_{hs} \qquad \forall\, h, i = 1, \ldots, n \quad \forall\, s, t = 1, \ldots, T \\ &\quad \boldsymbol{\beta}'_{it} \geq 0 \qquad \forall\, i = 1, \ldots, n \quad \forall\, t = 1, \ldots, T, \end{aligned} \tag{11}$$

where ϵ_{it} represent the residuals of the regression in time period t. The parameters α_{it} and $\boldsymbol{\beta}'_{it}$ characterize the tangent hyperplanes of the estimated production, which are specific to each unit in each time period. Thus, the frontier is estimated with as many as nT hyperplanes. The inequality constraints impose convexity using Afriat inequalities (Afriat, 1972), which are the key to modeling the concavity axiom in the general multiple regression setting (Kuosmanen, 2008).

[5]This problem is equivalent to the standard DEA model when a sign constraint on residuals is incorporated to the formulation ($\epsilon_i \leq 0, \forall i$) and considering the problem subject to shape constraints (monotonicity and convexity).

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Using the CNLS residuals from the first stage ($\hat{\epsilon}_{it}$), the next step is to estimate the expected value of the inefficiency term $\mu = E(u_i)$, thus we need to identify which part of the deviation from the frontier is attributable to inefficiency. For that purpose, and to maintain homogeneity of criteria with respect to the previously described SFA method, we also use the maximum likelihood estimation by following Equations (6) and (7). Analogously to the preceding subsection, in this second step firm-specific efficiencies are estimated by means of the Battese and Coelli (1988) point estimator, Equation (8), knowing now that $\hat{\epsilon_{it}} = \hat{\epsilon_{it}}^{CNLS} - \hat{\sigma}_u\sqrt{\frac{2}{\pi}}$ is the estimator of the composite error term.

The main limitation of this approach is that it is computational intensive, thus it might be difficult to apply when the sample size is large. However, several authors have proposed new algorithms to solve the CNLS formulation (e.g., Lee et al., 2013; Mazumder et al., 2019) with the aim of mitigating this drawback. Likewise, another possible limitation could be that, like the SFA models, this approach was originally designed for a production process with a single output. Nevertheless, it can be adapted to a multiple output framework using directional distance functions (Kuosmanen and Johnson, 2017) or ray production functions (Schaefer and Clermont, 2018).

4. Data and variables

In order to conduct this research, it was necessary to build a database about Panamanian hospitals, as there was no previous formal register including data on the activities and services and available resources for these institutions. This information was collected through a questionnaire designed specifically for the development of this research that was sent to the managers of all public hospitals that are part of the system. As we were interested in capturing information over a sufficiently long period of years, it was necessary to provide the support of expert staff in statistics and medical records in some cases, as the existing data were widely dispersed.

Our sample includes information about 22 public hospitals and 11 years (2005–2015), thus the total sample consists of 242 observations (22 hospitals $\times$ 11 years $=$ 242). These hospitals are divided between the two management systems that coexist in the country, the Social Security Fund (SSF) and the Ministry of Health (MoH). The former is an institution that offers healthcare to the insured population (approximately 75% of the population) and dependents through a network of comprehensive care services, while the MoH has the mission of ensuring access to health services for the entire population and the whole territory, including rural areas with more difficult access. In our sample, half of the hospitals (11) belong to each provider.

Our selection of variables was based on the input/output categories proposed by two comprehensive reviews of variables used in empirical studies focused on evaluating the efficiency of hospitals (O'Neill et al., 2008; Auteri et al., 2019), but also considering the limitations of the data collection process existing in the country. Specifically, we consider three inputs and two outputs to describe hospital production technology. As inputs, we selected the total number of beds as a proxy of capital and two variables representing human resources (medical and nonmedical staff). As for the outputs, although hospitals are recognized to provide different services, we restrict our analysis only to inpatients activities, as outpatient visits are carried by local health authorities. Moreover, it is also worth mentioning that we would have liked to include data about other

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Table 1
Descriptive statistics for total sample observations

Variables		Mean	S.D.	Min	Max
Outputs	Discharges	8885	8598	407	32,009
	Emergencies	52,153	36,082	2717	171,744
Inputs	Beds	205	209	15	843
	Medical staff	111	173	5	1021
	Nonmedical staff	321	256	6	1049

potential variables representing quality measures of hospitals' outcomes such as re-admissions or nosocomial infections, but this information was available only for a limited number of hospitals, thus we had to rule out their inclusion in our study. Therefore, we selected only two variables, the number of discharged patients and the number of emergency services, as output indicators.

Given the limited size of the available sample and the high correlation existing between the two outputs[6], we use the dimension reduction strategy based on factor analysis as suggested by Wilson (2018). This procedure reduces potential dimensionality related problems with nonparametric models and also facilitates the estimation of the SFA and StoNED models. The resulting output factor (FY) is determined by the first eigenvector of the second moment matrix of the two outputs, which can roughly be interpreted as an average of the scaled outputs. It explains 92% of total inertia, thus little information is lost by using this single output factor. Moreover, the correlation between this factor and the original output variables is also high (0.77 and 0.98 with discharges and emergencies, respectively). Therefore, in our model, we finally include three input variables and one output summarizing inpatient activities carried out by hospitals.

Table 1 shows the main descriptive statistics for the whole sample, that is, for the 242 observations available. The high values of standard deviation reveal the existence of significant heterogeneity among hospitals, with very diverse sizes and wide variations in their resource endowment.

Table 2 shows the mean values of each variable for each year of the period studied. As a complement to this information, Fig. 1 illustrates the evolution experienced by all these variables over the 11-year period under analysis. Here we can see that the evolution of the number of discharges has been on a downward trend since 2007. This phenomenon may be due in part to the growth of chronic diseases, which causes bed rotation to slow down and extend the period of hospitalization of patients, affecting the possibility of new admissions and reducing hospital discharges. In contrast, emergency services maintained an upward trend until 2012 and then declining in the last years of the period under study. With regard to the inputs, the volume of medical personnel is much lower than that of other workers. This gap has been maintained throughout the period studied, during which both staff numbers have increased significantly. The number of beds has remained constant over the 11 years studied in most hospitals, with only slight changes in some hospitals.

[6]The correlation coefficient is 0.61 and statistically significant.

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Table 2
Average values of each of the variables included in the model (2005–2015)

	Variables	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	Percentage change (2005-2015)
Outputs	Discharges	8749	9028	9288	9076	8925	9131	8,821	8,906	8,623	8,642	8,552	–2.25%
	Emergencies	47,519	50,050	51,085	50,930	50,781	56,178	54,008	56,743	51,911	53,216	51,268	7.89%
Inputs	Beds	202	203	203	204	204	204	205	206	207	206	207	2.48%
	Medical staff	95	98	101	103	103	105	108	125	126	131	130	36.84%
	Nonmedical staff	273	278	284	291	301	320	327	347	353	358	396	45.05%

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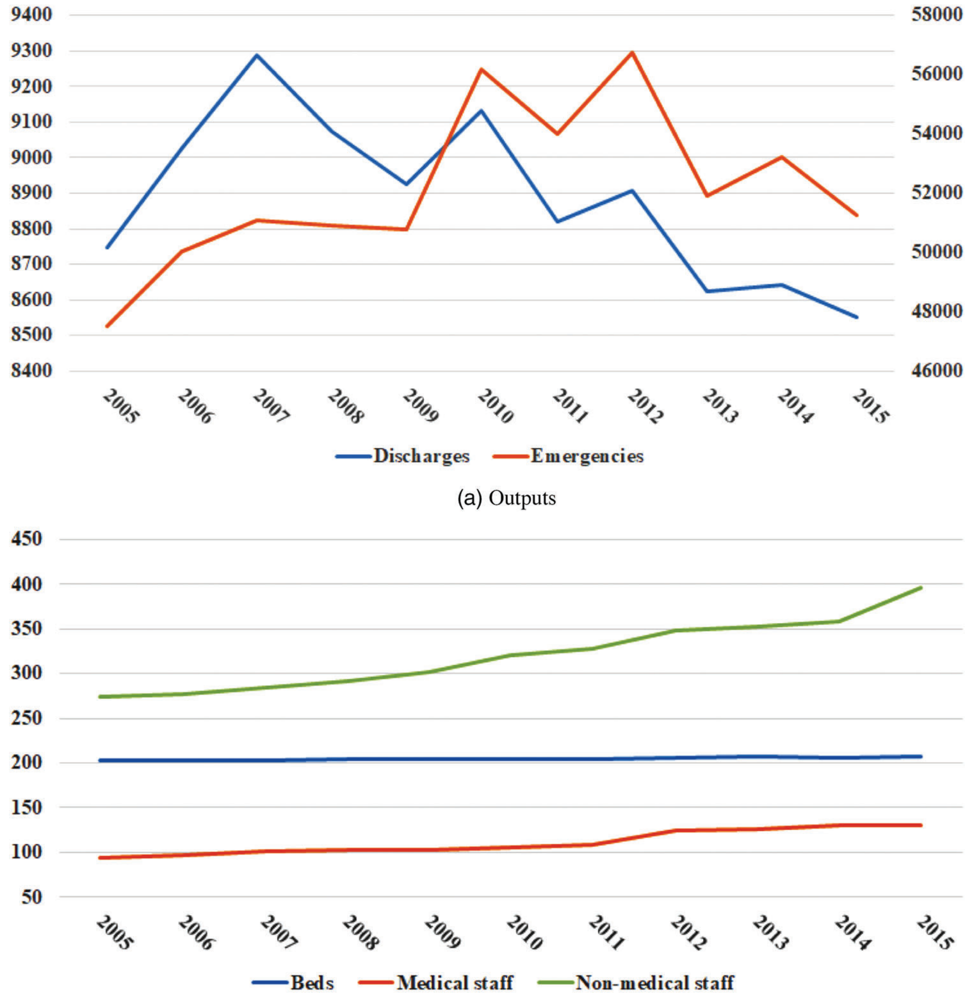

(a) Outputs

(b) Intputs

Fig. 1. Evolution of average values over the period.

5. Results

This section presents the efficiency scores estimated by applying the three methods described in Section 3 (DEA, SFA, and StoNED) to our data about public hospitals operating in Panama. As explained earlier in the text, the main difference between those approaches is that DEA is deterministic, that is, this method does not distinguish between inefficiency and noise, while SFA and StoNED do include a random noise component, which represents potential measurement errors or the effect of omitted explanatory variables.

Table 3 summarizes the main descriptive statistics of all the units evaluated in a dynamic context, that is, 22 hospitals over an 11-year period (2005–2015), which make a total of 242 observations.

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Table 3
Descriptive statistics of efficiency scores estimated with alternative methods

Method	Mean	S.D.	Min	Max
DEA	0.7088	0.2341	0.1210	1.0000
SFA	0.6095	0.1975	0.1790	0.9145
StoNED	0.7112	0.1296	0.2531	0.9448

Table 4
Correlation coefficients between different efficiency estimates

(a) Pearson's correlation coefficient

	DEA	**SFA**	**StoNED**
DEA	1		
SFA	0.7273	1	
StoNED	0.8853	0.8168	1

(b) Spearman's rank correlation coefficient

	DEA	**SFA**	**StoNED**
DEA	1		
SFA	0.6633	1	
StoNED	0.7998	0.7838	1

(c) Kendall's Tau correlation coefficient

	DEA	**SFA**	**StoNED**
DEA	1		
SFA	0.5266	1	
StoNED	0.6237	0.6141	1

(*) (All p-values are approximately 0.000 with a level of 0.1% of significance).

As can be seen, the average values of the efficiencies estimated with alternative approaches are quite similar for DEA and StoNED (both are close to 0.71), while the mean value of the efficiency scores estimated with the SFA approach are significantly lower (0.61). It is therefore clear that there is plenty of room for Panamanian hospitals to improve their efficiency levels. Likewise, it is also possible to verify that some fully efficient hospitals ($\theta = 1$) can be identified with the DEA method, while efficiencies estimated with the other two methods are all below one.

In order to check to what extent there are similarities between the efficiency measures estimated with alternative methods, we rely on two indicators. First, we calculated three different correlation coefficients (Pearson, Spearman and Kendall), whose values are reported in Table 4. According to these values, it seems that there are many similarities between the efficiency levels estimated with the different methods, as all of them are above 0.5 and are statistically significant. However, the probability density distributions of the estimated efficiencies displayed in Fig. 2 reveal that there are some notable divergences among them[7]. Specifically, we can observe that the distribution of

[7]These distributions have been obtained using nonparametric kernel density methods.

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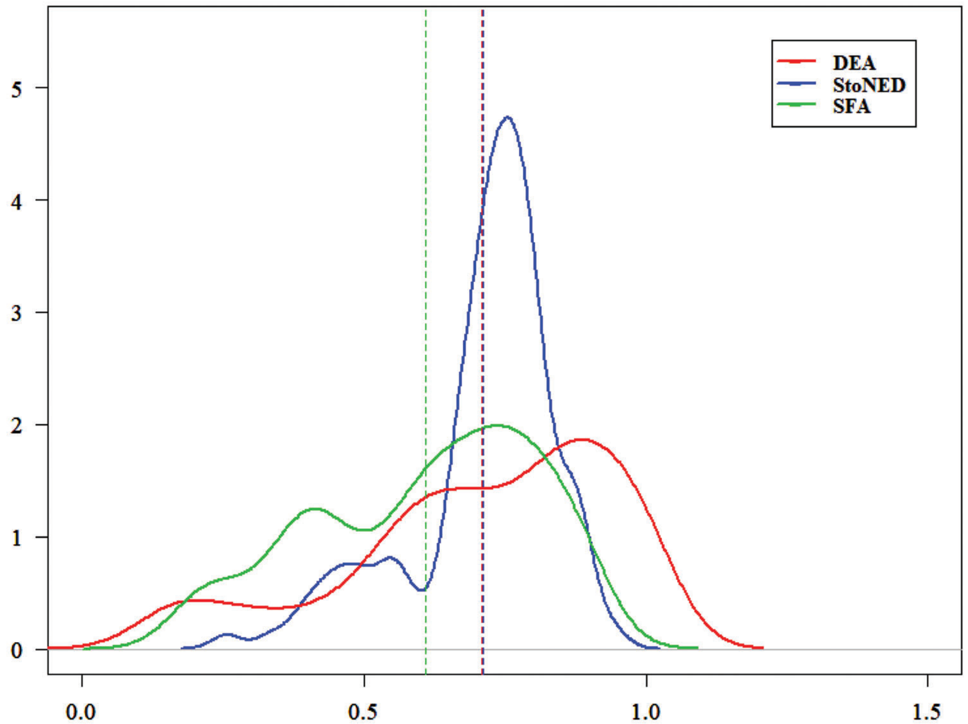

Fig. 2. Probability density of efficiencies estimated with alternative methods.

Table 5
Nonparametric tests for equality of distributions

Methods	Test statistic	*p*-value
DEA–SFA	11.1197	0.00***
DEA–StoNED	25.0726	0.00***
SFA–StoNED	12.7056	0.00***

scores estimated with StoNED presents a high proportion of units with values around the mean, represented by the vertical line, while the other two distributions are much more flattened. In both cases, a bimodal-like structure can be observed, although much more declined toward the right in the case of the efficiencies estimated with DEA as many of them are labeled as efficient, while the values estimated with SFA are all lower than one. Thus, in order to test whether those divergences between distributions are significant, we apply the nonparametric test proposed by Li et al. (2009)[8]. According to the values of the *p*-values displayed in Table 5, we can reject the null hypothesis that the distributions of the efficiency estimates can be considered as equal to each other in all cases, which corroborates that the observed divergences are significant.

As one of the main interests of this work is to analyze the efficiency of hospitals in a dynamic context, Fig. 3 shows the temporal evolution of the average efficiency scores estimated with each

[8]The test is based on a consistent integrated squared difference nonparametric test for equality of densities.

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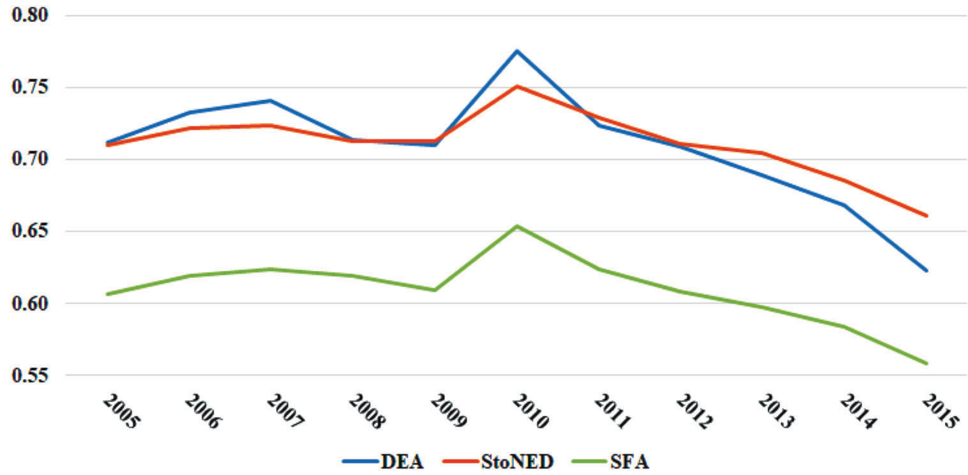

Fig. 3. Evolution of efficiency scores estimated with different approaches.

Table 6
Descriptive statistics of efficiency scores by management system

Efficiency model	Management model	Mean	S.D.	Min	Max	Kruskal Wallis test	Li et al. (2009) test
DEA	SSF	0.7083	0.2088	0.1210	1.0000	0.34	0.00***
	MoH	0.7093	0.2578	0.1436	1.0000		
SFA	SSF	0.5649	0.1800	0.1790	0.9024	0.00***	0.00***
	MoH	0.6542	0.2047	0.1972	0.9145		
StoNED	SSF	0.7015	0.1183	0.2531	0.9448	0.00***	0.00***
	MoH	0.7210	0.1398	0.3335	0.8940		

method over the 2005–2015 period. In all cases, it can be observed that there is a clear downward trend from 2010 onward. In this regard, it should be noted that between 2009 and 2014 the government declared public investments of more than 13 billion U.S. dollars to improve the infrastructure of several hospitals. In addition, the health payroll increased dramatically because of increases established by the wage laws governing healthcare employees as well as the payments for shifts and surgeries performed outside regular hours. All this resulted in an increase in the level of inputs that might explain the decrease of efficiency scores for those years.

After analyzing the results for the total set of hospitals, we focus on the comparison between hospitals belonging to each of the two management systems co-existing in the country (SSF and MoH). Table 6 displays the main descriptive statistics of efficiency scores estimated with alternative approaches for each group of hospitals. It can be seen that the average efficiencies of the Ministry's hospitals is higher than those for the hospitals belonging to the SSF, although these differences are only significant for the estimates made with SFA and StoNED, as can be seen from the *p*-values of the nonparametric Kruskal Wallis test also reported in the table[9]. Nevertheless, when we

[9]We have also applied an alternative nonparametric test (Mann–Whitney) and the results are very similar.

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explore the divergences existing between management models with regard to the distributions of efficiency scores, the p-values of the nonpametric test proposed by Li et al. (2009) suggest that they are significant for all the estimates.

Figure 4 shows the evolution of efficiency scores estimated with each method differentiating between hospitals belonging to each system. These graphs allow us to detect that the above-mentioned downward trend in efficiency levels is mainly explained by the decrease in efficiency levels experienced by the SSF hospitals, whereas the Ministry's hospitals maintain fairly stable levels of efficiency throughout the period studied. This is not surprising, as most of the previously mentioned increases in employee salary increases occurred mostly in SSF hospitals.

Finally, we explore the performance of hospitals on an individual basis. To do this, we rely on the information reported in Tables 7–9, which provide the efficiency scores estimated with DEA, SFA, and StoNED, respectively, for each hospital in different years. As hospitals are ranked according to the average values recorded throughout the period, it should be noted that the ranking of units are different depending on the methodology applied, especially with regard to the top positions in the ranking. Thus, for example, Hospital Dr. Aquilino Tejeira is identified as the top performer according to the classification of the average efficiencies estimated with SFA and StoNED, but it ranks third according to DEA calculations. Something similar occurs with hospital San Miguel Arcángel, placed at second best position according to SFA and StoNED, but fifth in the DEA rank. Even more striking is the fact that the hospital considered to be the most efficient according to the DEA model (Chiriquí Grande) is in the lower half of the ranking according to the other two methods. However, it is also possible to find many similarities between the rankings, which largely explain the high correlation found for the efficiency scores estimated with alternative approaches, mainly with regard to the identification of the worst performers. Thus, it can be seen that the hospitals placed at the bottom are practically the same in all three rankings (Hospital de la Palma, Hospital Azuero Anita Moreno, Hospital Dr. Rafael Hernández, Hospital Ezequiel Abadía and Complejo Hospitalario Arnulfo Arias Madrid).

When examining the evolution experienced by some hospitals over the period studied, some interesting aspects can also be observed. For instance, there are several hospitals (e.g., Changuinola, Dr. Rafael Hernández, Arnulfo Arias Madrid or Santo Tomás) in which there has been a very pronounced decrease in their efficiency levels estimated with alternative methods over the period studied. Thus, it can be assumed that the management of hospitalized patients has suffered a certain setback in recent years in those centers. In contrast, other hospitals like Dr. Cecilio A. Astillero or Dr. Joaquín Pablo Franco have experienced notable improvement over the years.

6. Conclusions

This study has implemented and compared three alternative approaches that can be used to estimate efficiency measures of performance with panel data. We compare the results obtained with the DEA and SFA methods, which have been widely applied in previous studies in the healthcare sector, with the estimates obtained by applying a much more innovative technique, the StoNED approach, which has hardly been used so far in this framework. The empirical analysis presented is referred to a sample of public hospitals operating in Panama, a country where this type of studies have not been conducted previously, and covers a period of 11 years (2005–2015) where there were

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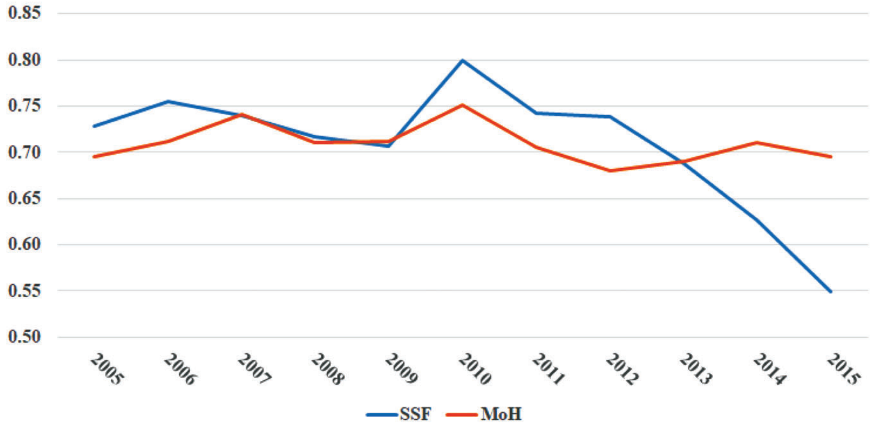
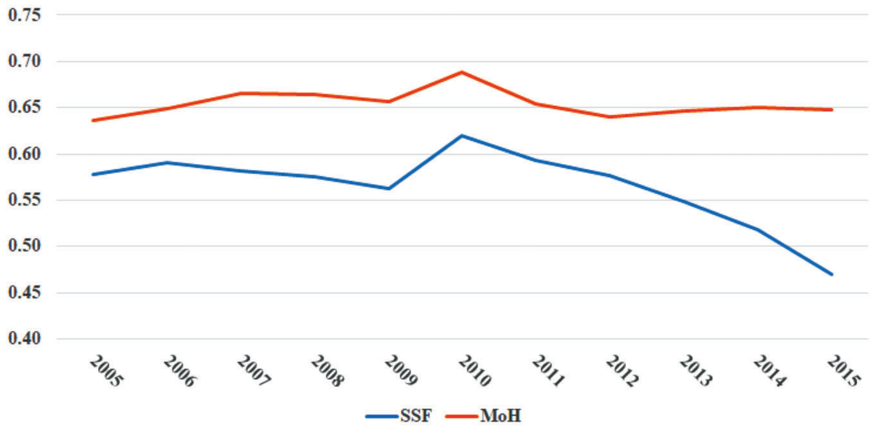
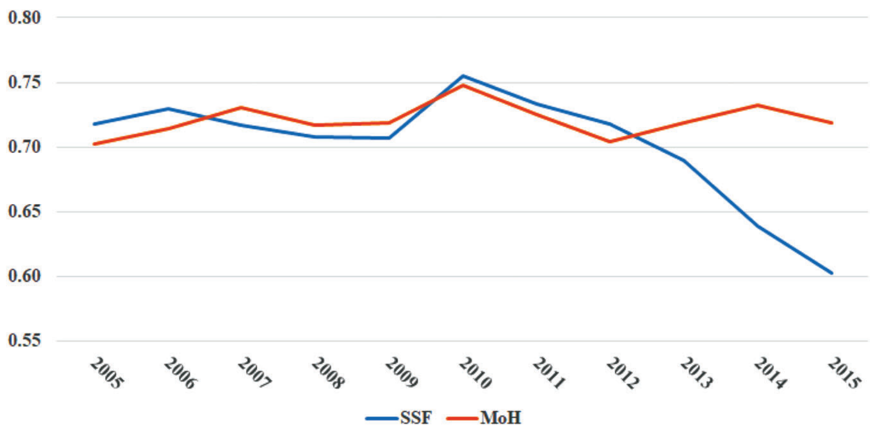


(a) Efficiency scores estimated with DEA



(b) Efficiency scores estimated with SFA



(c) Efficiency scores estimated with StoNED

Fig. 4. Evolution of efficiency scores by management system.

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Table 7
Evolution of efficiency scores for each hospital and each year (2005–2015) according to the DEA model

Hospital	Avg.	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Hospital de Chiriqui Grande	0.9393	1.0000	0.9340	0.8739	0.9241	0.9594	0.9719	0.9163	0.8389	0.9727	0.9410	1.0000
Hospital Dra. Susana Jones Cano	0.9339	0.9736	0.9489	0.8906	0.8323	0.8208	1.0000	0.9798	0.9603	0.9148	0.9518	1.0000
Hospital Dr. Aquilino Tejeira	0.9246	0.8782	0.8164	0.9167	0.9110	0.9147	0.9810	0.8979	0.8978	0.9566	1.0000	1.0000
Hospital Del Niño	0.9090	0.8658	1.0000	0.9913	0.9681	0.9548	0.9868	0.8301	0.8799	0.8003	0.8759	0.8463
Hospital San Miguel Arcangel	0.8911	1.0000	1.0000	0.9679	0.8602	0.9036	0.9471	0.8985	0.8888	0.8348	0.7777	0.7238
Hospital de Especialidades Pediatrica Omar Torrijos Herrera	0.8622	0.8728	0.9214	0.8338	0.7967	0.7209	0.8593	1.0000	0.9407	0.9192	1.0000	0.6198
Hospital Dr. Joaquin Pablo Franco	0.8336	0.5872	0.6149	0.6386	0.8774	0.9389	0.8683	0.8925	0.8617	0.9072	0.9831	1.0000
Hospital Materno Infantil Jose Domingo de Obaldia	0.8276	0.7054	0.7294	0.7976	0.8019	0.8598	0.9100	0.9243	0.8722	0.8538	0.8302	0.8187
Hospital Santo Tomas	0.7979	0.8484	0.8461	0.8421	0.8474	0.8176	0.8235	0.8092	0.7900	0.7291	0.7250	0.6984
Hospital Rafael H. Moreno	0.7932	0.9960	1.0000	1.0000	0.6770	0.6760	0.8243	0.7583	0.6859	0.7022	0.6859	0.7200
Hospital de Almirante	0.7774	0.8598	0.9690	1.0000	0.9329	0.9909	1.0000	0.6352	0.5114	0.6251	0.5356	0.4911
Hospital de Changuinola	0.7597	0.8852	0.8533	1.0000	0.8852	0.8533	1.0000	0.8852	0.8249	0.8852	0.1629	0.1210
Hospital Dr. Cecilio A. Castillero	0.7300	0.6825	0.6620	0.7586	0.7350	0.7213	0.7335	0.6752	0.5983	0.7845	0.8549	0.8246
Hospital Dr. Gustavo Nelson Collado (Chitre)	0.6741	0.6715	0.6880	0.6431	0.6255	0.5915	0.7954	0.6659	0.7742	0.7335	0.6433	0.5830
Policlinica Especializada Dr. Horacio Diaz Gomez	0.6647	0.5944	0.6386	0.6924	0.7440	0.5248	0.6110	0.6104	1.0000	0.6392	0.8307	0.4258
Hospital Luis Chicho Fabrega	0.6542	0.6561	0.7186	0.8009	0.7508	0.7023	0.6577	0.6138	0.5650	0.5520	0.5607	0.6181
Complejo Hospitalario Metropolitano Arnulfo A. Madrid	0.6140	0.6236	0.7476	0.7020	0.6356	0.6568	0.7151	0.6451	0.7471	0.4398	0.4262	0.4150
Hospital Dr. Rafael Estevez	0.6012	0.5435	0.5887	0.5342	0.5706	0.6024	0.7395	0.7925	0.5889	0.5650	0.5723	0.5160
Hospital Ezequiel Abadia	0.5227	0.4605	0.4785	0.4050	0.4103	0.5097	0.6179	0.6213	0.5688	0.5832	0.5263	0.5684
Hospital Regional Dr. Rafael Hernandez	0.4424	0.5302	0.5327	0.5638	0.5225	0.5494	0.4867	0.4187	0.3699	0.2861	0.3028	0.3034
Hospital Regional de Azuero Anita Moreno	0.2778	0.2510	0.2609	0.2587	0.2459	0.1824	0.3611	0.2768	0.2960	0.3144	0.3533	0.2547
Hospital de la Palma	0.1630	0.1736	0.1757	0.1802	0.1436	0.1637	0.1631	0.1773	0.1456	0.1583	0.1636	0.1480

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Table 8
Evolution of efficiency scores for each hospital and each year (2005–2015) according to the SFA model

Hospital	Avg.	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Hospital Dr. Aquilino Tejeira	0.8956	0.8813	0.8706	0.8823	0.8915	0.8949	0.9056	0.8966	0.8959	0.9064	0.9145	0.9117
Hospital San Miguel Arcangel	0.8464	0.8784	0.8784	0.8713	0.8407	0.8545	0.8663	0.8530	0.8500	0.8315	0.8071	0.7787
Hospital Dr. Joaquin Pablo Franco	0.7861	0.6438	0.6682	0.6880	0.7992	0.8235	0.8295	0.8414	0.7963	0.8397	0.8434	0.8739
Hospital de Especialidades Pediatrica Omar Torrijos Herrera	0.7688	0.7811	0.8034	0.7614	0.7413	0.6903	0.7758	0.8323	0.8157	0.8037	0.8364	0.6156
Hospital de Almirante	0.7680	0.8652	0.8975	0.9024	0.8648	0.8803	0.8794	0.7010	0.5871	0.6927	0.6112	0.5663
Hospital Dr. Cecilio A. Castillero	0.7627	0.7316	0.7165	0.7796	0.7659	0.7702	0.7589	0.7140	0.6816	0.8054	0.8386	0.8277
Hospital Dra. Susana Jones Cano	0.7469	0.7724	0.7690	0.7312	0.7031	0.6939	0.8001	0.7908	0.7499	0.7191	0.7363	0.7505
Hospital Del Niño	0.7414	0.7362	0.7980	0.7946	0.7886	0.7792	0.7927	0.7124	0.7322	0.6639	0.6870	0.6705
Hospital Dr. Gustavo Nelson Collado (Chitre)	0.7141	0.7385	0.7440	0.7060	0.6935	0.6559	0.7603	0.7205	0.7809	0.7503	0.6823	0.6225
Hospital Materno Infantil Jose Domingo de Obaldia	0.7138	0.6434	0.6611	0.7074	0.7160	0.7497	0.8056	0.7371	0.7170	0.7001	0.6940	0.7205
Hospital Luis Chicho Fabrega	0.7072	0.7021	0.7472	0.7947	0.7815	0.7421	0.7156	0.6806	0.6494	0.6504	0.6419	0.6740
Hospital Dr. Rafael Estevez	0.6285	0.5892	0.6278	0.5898	0.6242	0.6387	0.7430	0.7688	0.6126	0.5891	0.5942	0.5360
Hospital Rafael H. Moreno	0.5978	0.6203	0.6224	0.6242	0.5898	0.5501	0.6516	0.6067	0.5786	0.5788	0.5533	0.6001
Policlinica Especializada Dr. Horacio Diaz Gomez	0.5947	0.5157	0.5463	0.6197	0.6635	0.5150	0.5843	0.5979	0.8309	0.6070	0.6475	0.4141
Hospital Santo Tomas	0.5853	0.6420	0.6329	0.6264	0.6230	0.6041	0.5936	0.5840	0.5691	0.5297	0.5264	0.5067
Hospital Ezequiel Abadia	0.4957	0.4579	0.4734	0.4277	0.4306	0.5021	0.5625	0.5614	0.5052	0.5299	0.4869	0.5147
Hospital de Chiriqui Grande	0.4203	0.4731	0.4168	0.3933	0.4140	0.4056	0.4377	0.4160	0.3812	0.4360	0.3984	0.4515
Hospital Regional Dr. Rafael Hernandez	0.3766	0.4167	0.4198	0.4414	0.4352	0.4557	0.4371	0.3779	0.3241	0.2890	0.2688	0.2770
Hospital de Changuinola	0.3506	0.3814	0.3688	0.4266	0.3814	0.3688	0.4266	0.3814	0.3575	0.3814	0.2037	0.1790
Complejo Hospitalario Metropolitano Arnulfo A. Madrid	0.3493	0.3620	0.4226	0.4015	0.3694	0.3783	0.4110	0.3802	0.3997	0.2420	0.2393	0.2367
Hospital Regional de Azuero Anita Moreno	0.3456	0.3039	0.3154	0.3196	0.3048	0.2314	0.4386	0.3417	0.3704	0.3921	0.4270	0.3570
Hospital de la Palma	0.2143	0.2168	0.2238	0.2325	0.1986	0.2201	0.2138	0.2273	0.1995	0.2118	0.2161	0.1972

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Table 9
Evolution of efficiency scores for each hospital and each year (2005–2015) according to the StoNED model

Hospital	Avg.	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Hospital Dr. Aquilino Tejeira	0.8804	0.8725	0.8608	0.8774	0.8791	0.8805	0.8902	0.8745	0.8793	0.8876	0.8940	0.8892
Hospital San Miguel Arcangel	0.8313	0.8585	0.8585	0.8521	0.8259	0.8375	0.8477	0.8362	0.8337	0.8183	0.7989	0.7769
Hospital de Almirante	0.8272	0.9232	0.9435	0.9448	0.8917	0.8979	0.8802	0.7725	0.6921	0.7672	0.7106	0.6754
Hospital Rafael H. Moreno	0.8248	0.8728	0.8735	0.8737	0.8121	0.7882	0.8403	0.8205	0.7912	0.7982	0.7912	0.8107
Hospital Dr. Gustavo Nelson Collado (Chitre)	0.7901	0.8050	0.8093	0.7848	0.7760	0.7572	0.8513	0.8141	0.8315	0.8012	0.7524	0.7083
Hospital Del Niño	0.7848	0.7654	0.8096	0.8072	0.8022	0.7986	0.8095	0.7578	0.7810	0.7494	0.7803	0.7717
Hospital Santo Tomas	0.7636	0.7683	0.7732	0.7773	0.7808	0.7745	0.7758	0.7739	0.7678	0.7401	0.7397	0.7285
Hospital Dr. Joaquin Pablo Franco	0.7524	0.6308	0.6504	0.6672	0.7591	0.7815	0.7988	0.8078	0.7516	0.7976	0.7972	0.8346
Hospital Dra. Susana Jones Cano	0.7493	0.7653	0.7660	0.7360	0.7188	0.7111	0.7979	0.7931	0.7488	0.7246	0.7367	0.7442
Hospital de Especialidades Pediatrica Omar Torrijos Herrera	0.7405	0.7417	0.7616	0.7246	0.7077	0.6657	0.7369	0.7888	0.7833	0.7741	0.8036	0.6580
Hospital Materno Infantil Jose Domingo de Obaldia	0.7392	0.6700	0.6839	0.7196	0.7263	0.7519	0.7789	0.7732	0.7690	0.7531	0.7550	0.7504
Hospital Dr. Cecilio A. Castillero	0.7376	0.7056	0.6931	0.7458	0.7340	0.7453	0.7351	0.6982	0.6759	0.7765	0.8068	0.7974
Hospital de Chiriqui Grande	0.7316	0.7945	0.7386	0.7092	0.7308	0.7291	0.7385	0.7124	0.6846	0.7441	0.7218	0.7438
Policlinica Especializada Dr. Horacio Diaz Gomez	0.7217	0.6606	0.6877	0.7417	0.7704	0.6680	0.7213	0.7330	0.8680	0.7370	0.7634	0.5876
Hospital Dr. Rafael Estevez	0.7114	0.6818	0.7180	0.6646	0.6919	0.7252	0.7950	0.8118	0.7075	0.6882	0.6928	0.6483
Hospital Luis Chicho Fabrega	0.7016	0.6916	0.7280	0.7676	0.7598	0.7256	0.7060	0.6783	0.6674	0.6678	0.6545	0.6711
Complejo Hospitalario Metropolitano Arnulfo A. Madrid	0.6808	0.6776	0.7510	0.7316	0.6999	0.7131	0.7547	0.7199	0.7778	0.5650	0.5537	0.5444
Hospital de Changuinola	0.6396	0.7212	0.7069	0.7654	0.7212	0.7069	0.7654	0.7212	0.6933	0.7212	0.2594	0.2531
Hospital Ezequiel Abadia	0.6334	0.5928	0.6091	0.5245	0.5299	0.6339	0.7135	0.7138	0.6722	0.6771	0.6333	0.6677
Hospital Regional Dr. Rafael Hernandez	0.4907	0.5325	0.5353	0.5592	0.5485	0.5692	0.5495	0.4880	0.4368	0.3839	0.3952	0.3998
Hospital Regional de Azuero Anita Moreno	0.4666	0.4297	0.4427	0.4420	0.4251	0.3376	0.5718	0.4678	0.4948	0.5181	0.5652	0.4374
Hospital de la Palma	0.4486	0.4634	0.4817	0.5018	0.3852	0.4892	0.4701	0.4903	0.3335	0.4007	0.4768	0.4415

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important changes in the allocation of resources for these public healthcare providers. The main results can be summarized as follows.

Our results suggest that there has been a decrease in the efficiency levels of Panamanian hospitals throughout the period studied, and that there is still ample room for improvement for most of them. This evidence is observed for all estimates made with alternative models. In fact, we find little variation in the values of efficiency scores estimated with both nonparametric alternatives (DEA and StoNED), being much lower for SFA. In contrast, when examine the probability density distributions of the efficiencies, we observe greater similarities between scores estimated with DEA and SFA. In any case, the rankings of hospitals derived from different approaches are quite similar, especially with regard to the identification of the worst performers, thus we can conclude that the estimates of the most innovative model applied in this empirical research, the StoNED approach, are in line with the efficiency scores obtained with standard traditional models like DEA and SFA.

A detailed analysis of the differences existing between the hospitals belonging to the two public management systems that coexist in the country has allowed us to identify that the efficiencies of Ministry's hospitals are higher than those for the hospitals belonging to the Social Security Fund. Likewise, the analysis of the evolution of efficiencies throughout the period has allowed us to verify that the divergences between the hospitals belonging to each management scheme arise mainly in the last years of the period, in which the notable increase experienced by the SSF hospitals' expenses has not been reflected in an improvement of the results, causing the corresponding decrease in their efficiency levels.

Despite these interesting results, we should mention that we are aware that our study presents a series of limitations that should lead us to interpret them with caution. Probably the most important of these limitations is that we have not been able to include in our empirical analysis some variables that may be relevant due to the absence of data. These include the use of some indicator of the quality of health services, some type of complexity-adjustment (e.g., case-mix index), or some other environmental variable representative of the typology of the patients in each hospital, such as the percentage of the population above 65 years old living in its area of influence. Unfortunately, this is a common problem in studies conducted in developing countries (Hafidz et al., 2018), although, in our opinion, this should not be an obstacle to conduct assessments in this framework, because only through them it is possible to identify benchmark units and organizations that are not maximizing their resources, which should introduce changes to improve their performance. In any case, it is worth mentioning that our results should not be used to make very strong policy recommendations such as closing hospitals or reducing investments in poorly performing institutions, especially because apart from efficiency, there are other relevant health-system goals like access or equity of care that have not been considered in the present study.

Another serious problem of our study is that the number of observations in our sample is quite small, as it only includes 22 hospitals. However, we should mention that the hospitals that make out our sample represent approximately 70% of total beds in the public hospital system and around 80% of the personnel, thus we consider our sample to be fairly representative of the total number of hospitals in the Panamanian public health system.

Finally, it is worth mentioning that the design of our study has been largely conditioned because two of the techniques used to measure efficiency were designed to be implemented in a single-output framework (SFA and StoNED). However, it is worth mentioning that this potential limitation can be overcome by using applying directional distance functions, which allows for handling multiple

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outputs in the estimation of efficiency scores. Therefore, a potential extension of the present study could be to check whether shifting from one-output to two-output in the specification or our production technology might affect the results.

Acknowledgments

The authors acknowledge the helpful comments received from Daniel Santín (Complutense University) and Julián Ramajo (University of Extremadura). Likewise, Jose M. Cordero and Cristina Polo thank the financial support from the Junta de Extremadura through grants IB16171 and GR18106. Finally, Agustín García thanks the financial support from the Junta de Extremadura through grant GR18075.

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