

Canonical Symplectic Structures on the r -th Order Tangent Bundle of a Symplectic Manifold

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1. ABSTRACT

We describe all canonical 2-forms $\Lambda(\omega)$ on the r -th order tangent bundle $T^r M = J_0^r(\mathbf{R}, M)$ of a symplectic manifold (M, ω) . As a corollary we deduce that all canonical symplectic structures $\Lambda(\omega)$ on $T^r M$ over a symplectic manifold (M, ω) are of the form $\Lambda(\omega) = \sum_{k=0}^r \alpha_k \omega^{(k)}$ for all real numbers α_k with $\alpha_r \neq 0$, where $\omega^{(k)}$ is the (k) -lift (in the sense of A. Morimoto) of ω to $T^r M$.

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