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Hereditarily Normaloid Operators

B. P. Duggal *

8 Redwood Grove, Northfields Avenue, Ealing, London W5 4SZ, *United Kingdom*
e-mail: bpduggal@yahoo.co.uk

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Abstract

A Banach space operator $T \in B(\mathcal{X})$ is said to be *hereditarily normaloid*, $T \in \mathcal{H}N$, if every part of T is normaloid; $T \in \mathcal{H}N$ is *totally hereditarily normaloid*, $T \in \mathcal{T}HN$, if every invertible part of T is also normaloid; and $T \in \mathcal{C}HN$ if either $T \in \mathcal{T}HN$ or $T - \lambda I$ is in $\mathcal{H}N$ for every complex number λ. Class $\mathcal{C}HN$ is large; it contains a number of the commonly considered classes of operators. We study operators $T \in \mathcal{C}HN$, and prove that the Riesz projection associated with a $\lambda \in \text{iso}\sigma(T)$, $T \in \mathcal{C}HN \cap B(\mathcal{H})$ for some Hilbert space $\mathcal{H}$, is self-adjoint if and only if $(T - \lambda I)^{-1}(0) \subseteq (T^* - \overline{\lambda} I)^{-1}(0)$. Operators $T \in \mathcal{C}HN$ have the important property that both T and the conjugate operator T^* have the *single-valued extension property* at points λ which are not in the *Weyl spectrum* of T; we exploit this property to prove *a-Browder* and *a-Weyl theorems* for operators $T \in \mathcal{C}HN$.

* To Professor Carl Pearcy on his seventieth birthday.

Key words: Banach space, Weyl's theorem, single valued extension property, hereditarily normaloid operators, paranormal and $*$-paranormal operators.