EXTRACTA MATHEMATICAE Vol. **21**, Núm. 2, 107 – 147 (2006)

# Non-trivial Derivations on Commutative Regular Algebras

A.F. Ber[1], V.I. Chilin[1], F.A. Sukochev[2]

[1] *Department of Mathematics, National University of Uzbekistan Tashkent, Uzbekistan*

[2] *School of Informatics and Engineering, Flinders University Bedford Park, SA 5042, Australia*

*e-mail: chilin@ucd.uz, sukochev@infoeng.flinders.edu.au*

(Presented by Willian B. Jonhson)

AMS *Subject Class.* (2000): 46H40, 46L51, 46L57 *Received May* 15, 2006

## 1. Abstract

Necessary and sufficient conditions are given for a (complete) commutative algebra that is regular in the sense of von Neumann to have a non-zero derivation. In particular, it is shown that there exist non-zero derivations on the algebra $L(M)$ of all measurable operators affiliated with a commutative von Neumann algebra $M$, whose Boolean algebra of projections is not atomic. Such derivations are not continuous with respect to measure convergence. In the classical setting of the algebra $S[0,1]$ of all Lebesgue measurable functions on $[0,1]$, our results imply that the first (Hochschild) cohomology group $H^1(S[0,1], S[0,1])$ is non-trivial.

## References

[1] Ayupov, Sh.A., Topology of $C$-convergence in semifields, *Izv. Akad. Nauk UzSSR Ser. Fiz.-Mat. Nauk* 1976, no. 5, 3 – 7, 83.

[2] Ayupov, Sh.A., Derivations in algebras of measurable operators, *Dokl. Uzbek Akad. Nauk* **3** (2000), 14 – 17.

[3] Ber, A.F. , Sukochev, F.A., Chilin, V.I., Derivations in commutative regular algebras, *Math. Notes* **75** (2004), 418 – 419.

[4] Bratelli, O., Robinson, D.W., "Operator Algebras and Quantum Statistical Mechanics", Vol. 1, Springer-Verlag, New York, 1979.

[5] Cartan, H., Eilenberg, S., "Homological Algebra", Princeton University Press, Princeton, NJ, 1956.

[6] Clifford, A.N., Preston, G.B., "The Algebraic Theory of Semigroup", Vol. 1, American Mathematical Society, Princeton, RI, 1964.

[7] Dales, H.G., "Banach Algebras and Automatic Continuity", Clarendon Press, Oxford, 2000.

[8] Chilin, V.I., Partially ordered Baer involutive algebras, in "Current Problems in Mathematics, Newest Results", Vol. 27, 99 – 128, Akad. Nauk SSSR, Vsesoyuz. Inst. Nauchn. i Tekhn. Inform., Moscow, 1985 (Russian),.

[9] Inoue, A., Ôta, S., Derivations on algebras of unbounded operators, *Trans. Amer. Math. Soc.* **261** (1980), 567 – 577.

[10] Inoue, A., Ôta, S., Tomiyama, J., Derivations of operator algebras into space of unbounded operators, *Pacific J. Math.* **96** (1981), 389 – 404.

[11] Halmos, P.R., "Measure Theory", D. Van Nostrand Company, Inc., New York, 1950.

[12] Maeda, F., "Kontinuierliche Geometrien", Springer-Verlag, Berlin, 1958.

[13] Sakai, S., "$C^*$–Algebras and $W^*$–Algebras", Springer-Verlag, New York, 1971.

[14] Sakai, S., "Operator Algebras in Dynamical Systems. The Theory of Unbounded Derivations in $C^*$–Algebras", Cambridge University Press, Cambridge, 1991.

[15] Segal, I.E., A non-commutative extension of abstract integration, *Ann. of Math.* **57** (1953), 401 – 457.

[16] Skornjakov, L.A., "Complemented Modular Lattices and Regular Rings", Gosudarstv. Izdat. Fiz.-Mat. Lit., Moscow, 1961 (Russian).

[17] Takesaki, M., "Theory of Operator Algebras I", Springer-Verlag, New York, 1979.

[18] Vladimirov, D.A., "Boolean Algebras", Nauka, Moscow, 1969.

[19] van der Waerden, B.L., "Algebra I", Springer-Verlag, New York, 1971.

[20] Yeadon, F.J., Convergence of measurable operators, *Proc. Camb. Phil. Soc.* **74** (1973), 257 – 268.