EXTRACTA MATHEMATICAE Vol. **21**, Núm. 2, 93 – 106 (2006)

On the Moore-Penrose Inverse in C^*-algebras

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(Presented by M. Mbekhta)

AMS *Subject Class.* (2000): 46L05 *Received November* 5, 2005

Abstract

In this article, two results regarding the Moore-Penrose inverse in the frame of C^*-algebras are considered. In first place, a characterization of the so-called reverse order law is given, which provides a solution of a problem posed by M. Mbekhta. On the other hand, Moore-Penrose hermitian elements, that is C^*-algebra elements which coincide with their Moore-Penrose inverse, are intro- duced and studied. In fact, these elements will be fully characterized both in the Hilbert space and in the C^*-algebra setting. Furthermore, it will be proved that an element is normal and Moore-Penrose hermitian if and only if it is a hermitian partial isometry.

References

[1] Andruchow, E., Corach, G., Differential geometry of partial isometries and partial unitaries, *Illinois J. Math.,* **48** (1) (2004), 97 – 120.

[2] Barwick, D.T., Gilbert, J.D., On generalizations of the reverse order law, *SIAM J. Appl. Math.,* **27** (2) (1974), 326 – 330.

[3] Barwick, D.T., Gilbert, J.D., Generalizations of the reverse order law with related results, *Linear Algebra Appl.,* **8** (1974), 345 – 349.

[4] Bouldin, R., The pseudo-inverse of a product, *SIAM J. Appl. Math.,* **24** (4) (1973), 489 – 455.

[5] Bouldin, R., Closed range and relative regularity for products, *J. Math. Anal. Appl.,* **61** (1977), 397 – 403.

[6] Conway, J.B., "A Course in Operator Theory", Graduate Studies in Mathematics, 21, American Mathematical Society, Providence, RI, 2000.

[7] Greville, T.N.E., Note on the generalized inverse of a matrix product, *SIAM Rev.,* **8** (4) (1966), 518 – 521, *Erratum, ibid* **9** (2) (1967), 249.

[8] Halmos, P.R., "A Hilbert Space Problem Book", Van Nostrand, New York, 1966.

[9] Harte, R., "Invertibility and Singularity for Bounded Linear Operators", Monographs and Textbooks in Pure and Applied Mahematics, 109, Marcel Dekker Inc., New York, 1988.

[10] Harte, R., Mbekhta, M., On generalized inverses in C^*-algebras, *Studia Math.,* **103** (1992), 71 – 77.

[11] Harte, R., Mbekhta, M., On generalized inverses in C^*-algebras II, *Studia Math.,* **106** (1993), 129 – 138.

[12] Izumino, S., The product of operators with closed range and an extension of the reverse order law, *Tôhoku Math. J.,* **34** (1982), 43 – 52.

[13] Khosravi, A., Alizadeh, M.H., Generalized inverses of products, *Int. J. Appl. Math.,* **10** (2) (2002), 141 – 148.

[14] Mbekhta, M., Conorme et inverse généralisé dans les C^*-algèbres, *Canadian Math. Bull.,* **35** (4) (1992), 515 – 522.

[15] Mbekhta, M., Partial isometries and generalized inverses, *Acta Sci. Math. (Szeged),* **70** (3-4) (2004), 767 – 781.

[16] Penrose, R., A generalized inverse for matrices, *Proc. Cambridge Philos. Soc.,* **51** (1955), 406 – 413.

[17] Shijie, L., The range and pseudo-inverse of a product, *Tôhoku Math. J.,* **89** (1987), 89 – 94.

[18] Shinozaki, N., Sibuya, M., The reverse order law $(AB)^- = B^-A^-$, *Linear Algebra Appl.,* **9** (1974), 29 – 40.