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# Maintenance cost evaluation for heterogeneous complex systems under continuous monitoring

Inma T. Castro\*

*Department of Mathematics  
University of Extremadura*

Rob J.I. Basten

*Department of Industrial Engineering and Innovation Sciences  
Eindhoven University of Technology*

Geert-Jan van Houtum

*Department of Industrial Engineering and Innovation Sciences  
Eindhoven University of Technology*

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## Abstract

The maintenance strategy for a complex system consisting of both non-monitored and monitored components is analyzed in this paper. Non-monitored components can only be maintained correctively upon failure. Monitored components are monitored continuously and are maintained when they become too degraded, i.e., when its degradation level hits a threshold. For this complex system, an opportunistic maintenance strategy is implemented, meaning that a maintenance intervention for a component can be used as an opportunity for preventive maintenance of monitored components: If the degradation level of a monitored component exceeds a preventive threshold at the time of another maintenance intervention, this component is maintained preventively. By performing these maintenance actions, different costs are incurred. The main purpose of this paper is to evaluate the expected cost rate of the system. To that end, two methods are compared: renewal and semi-regenerative techniques. Using renewal techniques, the evaluation of the expected cost rate of this maintenance

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\*Corresponding author. Email: [inmatorres@unex.es](mailto:inmatorres@unex.es)

strategy is time consuming, especially for a large number of monitored components. However, using semi-regenerative techniques the required computation time is drastically shortened: For a system with ten monitored components, the computation time to evaluate the optimal maintenance strategy goes from more than a day to few seconds. Numerical examples are given to illustrate the results. The conclusion is that for a large number of monitored components, semi-regenerative techniques are more appropriate to evaluate the expected cost rate in terms of computation time.

*Keywords:* Condition-based maintenance, opportunistic maintenance, complex systems, semi-regenerative process

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## 1. Introduction

Condition-Based Maintenance (CBM) for complex technical systems is attracting a lot of attention in recent years due to the rapid development of sensor technology and the development of stochastic degradation models. Using information collected through condition monitoring, a maintenance action is scheduled just before a hard failure threshold is crossed [1]. This is a preventive maintenance action that is executed at short term, at a moment that is convenient for the user of the system and that avoids a hard failure and hence unplanned downtime. Such maintenance actions are referred to as *Just-In-Time (JIT) maintenance* actions.

Early work on CBM is focused on single-component systems. However, systems in practice are more and more complex, consisting of many components. For multi-component systems, the implementation of an opportunistic maintenance program along with CBM can reduce the maintenance costs. Opportunistic maintenance aims to perform preventive maintenance tasks at opportunities [2]. For example, whenever a failure happens in an offshore wind farm, the maintenance team performs a corrective maintenance action. To reduce the total maintenance set up cost, the maintenance team takes this corrective maintenance time as an opportunity to check the state of the rest of components and

to perform additional maintenance tasks if necessary [3]. In [4] the impact of opportunistic maintenance on the effectiveness of CBM is analyzed .

In complex systems, components can be subject to different mechanisms of failure, e.g., mechanical, hydraulic, or electronic. As a result, some components may fail suddenly without advance warning and other components may degrade gradually towards failure [5]. Monitoring cannot prevent the former kind of failures and implementation of a monitoring process is not possible for all components of this complex system [6],[7]. In case of sudden failures, a corrective maintenance policy is the only reasonable option. Because of the different types of failures and corresponding maintenance policies, maintenance coordination in a complex system with heterogeneous components is extremely difficult.

In this paper, a complex system consisting of  $m$  continuously monitored components and  $n$  non-monitored components is modelled. The monitored components are subject to continuous degradation following a gamma process, which is a common choice in probabilistic modeling ([8]:[10]). The monitored components are maintained condition-based; maintenance is performed on a component when its degradation level crosses the *just-in-time threshold*. The non-monitored components are subject to a corrective policy, that is, the replacement of such components is performed after failures occur. Assuming  $n$  large enough, the time between failures in the non-monitored components is exponentially distributed [6]. When maintenance is performed on a component, it provides an opportunity for maintenance on the (other) monitored components. If the degradation of such a monitored component exceeds a *preventive threshold*, the component is preventively maintained. Determining the optimal maintenance policy that leads to the lowest total expected cost per unit time means determining the preventive thresholds for all monitored-components. Evaluating a given policy using renewal technique is very complex and time-consuming. To deal with this, semi-regenerative techniques are used to evaluate the expected cost rate [11] . Semi-regenerative techniques have been used for maintenance optimization of systems before, mainly for single-item systems ([12]-[14]) and two-item systems ([15],[16]), and without also incorporating non-monitored

components. To the best of our knowledge, no comparison of the computation times using renewal and semi-regenerative techniques has been made.

In other papers, systems consisting of heterogeneous components are analyzed taking into account different maintenance policies for individual components ([6],[7],[17]). In [6] and [17], an approximate evaluation procedure has been developed for a multi-component system with one monitoring component and corrective maintenance (CM) on the other components of the same system. Our paper extends the number of monitoring components to  $m$ . In [7], an exact evaluation procedure is developed for a mix of components following condition-based and age/usage-based maintenance policies. The difference between [7] and our paper is that the former considers a delay time model to describe the degradation of the condition-based components while our paper uses stochastic processes to describe the degradation of the components.

Examples of systems consisting of heterogeneous components are found in multiple high-tech companies. For instance, an electric power distribution system consists of a capacitor and a transformer [18]. The capacitor is subject to gradual degradation whereas the transformer is stopped immediately when suffering superfluous damage. Litography systems are also examples of these complex and heterogeneous systems. Some components of a litography system are maintained condition-based while others are failure-based maintained [19]. Maintenance optimization of these heterogeneous systems is shown at a company that designs, builds, sells and maintains industrial printers. It is found that some components clearly show degradation, for example certain belts that transport paper inside the printer, or filters that slowly get clogged [20]. Others, especially electronic components, do not give any warning in advance, or at least the authors have not been able to recognize it. These components can only be replaced correctively. Finally, in [17], this modelling of heterogeneous systems with different maintenance policies was validated for a compressor.

In short, the main contributions of this paper are:

- Under the given policy structure with preventive thresholds for the moni-

tored components, an evaluation procedure for a complex system consisting of multiple monitored components plus non-monitored components is developed.

- Semi-regenerative techniques are applied and it is shown that the computation time of our evaluation procedure is very small (just a few seconds). The computation time is linear in the number of monitored components.
- It is also shown that the computation time of our procedure is much lower than the computation time using renewal theory. In that case, the computation time appears to grow exponentially in the number of monitored components.

This paper is organized as follows. In Section 2, the functioning of the system is modelled. Section 3 is devoted to the evaluation of the kernel of the semi-regenerative process used to describe the system functioning. Section 4 analyzes the expected cost rate under a given maintenance policy. In Section 5, the speed of our numerical evaluation procedure is investigated and comparisons with renewal theory techniques are performed. The use of our evaluation procedure within an optimization procedure for the preventive thresholds for all monitored components is also demonstrated. Finally, conclusions are shown in Section 6.

## 2. System description

Assumptions are first given in Section 2.1 and then the degradation behaviour is explained in Section 2.2.

### 2.1. Assumptions

A complex system consisting of  $m$  monitored components and  $n$  non-monitored components is analyzed.

1. An infinite time horizon  $[0, \infty)$  is assumed.
2. The  $m$  monitored components are subject to internal degradation. The degradation of the  $i$ -th component evolves according to a homogeneous

gamma process with parameters  $\alpha_i$  and  $\beta_i$  for  $i \in I^m$  where  $I^m = \{1, 2, \dots, m\}$ . Let  $X_i(t)$  be the degradation level of component  $i$  at time  $t$  with  $X_i(0) = 0$ . For  $s < t$ , the density of  $X_i(t) - X_i(s)$  is given by

$$f_{\alpha_i(t-s), \beta_i}(x) = \frac{\beta_i^{\alpha_i(t-s)}}{\Gamma(\alpha_i(t-s))} x^{\alpha_i(t-s)-1} e^{-\beta_i x}, \quad x \geq 0,$$

where  $\Gamma(\cdot)$  denotes the gamma function given by

$$\Gamma(\alpha_i(t-s)) = \int_0^\infty u^{\alpha_i(t-s)-1} e^{-u} du.$$

3. The monitoring process is continuous.
4. Monitored components are maintained according to a condition-based maintenance policy. When the degradation level of component  $i \in I^m$  crosses a failure threshold  $\tilde{L}_i$ , component  $i$  would fail (i.e., the component is then too degraded and works no longer properly). Just before, i.e., when the degradation level crosses a just-in-time threshold  $L_i$  ( $L_i < \tilde{L}_i$  and  $\tilde{L}_i - L_i$  is very small), a just-in-time maintenance action is performed with an associated cost of  $C_i^j$  monetary units with  $i \in I^m$ .
5. When a just-in-time maintenance action is performed on one monitored component, it provides an opportunity for the other monitored components to be maintained together. Let  $M_i$  be the preventive threshold for component  $i$ ,  $i \in I^m$ . At the time of the just-in-time maintenance action, the degradation levels of the rest of the monitored components are checked. If the degradation level of a component  $i'$  exceeds its preventive threshold  $M_{i'}$ , this component is preventively replaced. The associated cost of the preventive maintenance action of the  $i'$ -th monitored component is equal to  $C_{i'}^p$  monetary units. Also under a preventive maintenance action, the component is replaced by a new or ready-for-use one. For each component  $i \in I^m$ , we assume that  $C_i^p < C_i^j$ , because  $C_i^p$  represents the cost for replacing the current component only (i.e., the cost of the visit of a repairman to the system is already included in the just-in-time maintenance cost for another component).

6. Non-monitored components are subject to *sudden failures*. Let  $Y$  be the time between sudden failures with survival function,

$$\bar{F}_Y(t) = \exp(-\lambda t), \quad t \geq 0.$$

When a sudden failure arrives to the system, a corrective maintenance action is immediately performed on the non-monitored component

7. A just-in-time maintenance action, a corrective maintenance action and a preventive action mean that a repairman replaces the component by a new or ready-for-use one.
8. A corrective maintenance action implies an associated cost of  $C^f$  monetary units. The cost consists of the cost of a visit of the repairmen to the system, the cost of replacing the current component by a new or ready-for-use one, and the cost of the unplanned downtime between the occurrence of the failure and the repair completion. This unplanned downtime is often expensive and hence  $C^f$  is in general (much) larger than each of the  $C_i^j$ ,  $i \in I^m$ . The corrective maintenance action for the non-monitored component also provides an opportunity for the monitored components. It means that, if the degradation levels of any of these monitored components  $i \in I^m$  exceeds the preventive threshold when a sudden failure arrives to the system, a preventive maintenance of this component is performed with a cost of  $C_i^p$  monetary units.

In short, the following possibilities for maintenance actions exist:

- **Non-monitored components:**

- A failure in a non-monitored component leads to its corrective maintenance.

- **Monitored components:**

- If a monitored component is too degraded, i.e., its degradation level passes the just-in-time-threshold  $L_i$ , this component is just-in-time maintained.



- In case of corrective maintenance of a non-monitored component or just-in-time maintenance of a monitored component, (other) monitored components that have passed the preventive threshold are opportunistically maintained.

## 2.2. Degradation behaviour

When the degradation level of component  $i$  exceeds  $L_i$ , just-in-time maintenance is performed. Let  $\sigma_{L_i}$  be the time for component  $i \in I^m$  to reach degradation level  $L_i$ , then

$$\sigma_{L_i} = \inf \{t \geq 0, X_i(t) \geq L_i\}.$$

The distribution function of  $\sigma_{L_i}$  is given by [21]

$$F_{\sigma_{L_i}}(t) = \frac{\Gamma(\alpha_i t, L_i \beta_i)}{\Gamma(\alpha_i t)}, \quad t \geq 0, \quad i = 1, 2, \dots,$$

where  $\Gamma(\alpha_i t, L_i \beta_i)$  denotes the incomplete gamma function

$$\Gamma(\alpha_i t, L_i \beta_i) = \int_{L_i \beta_i}^{\infty} u^{\alpha_i t - 1} e^{-u} du.$$

Considering the threshold  $M_i$  ( $M_i < L_i$ ) the distribution of the variable  $\sigma_{L_i} - \sigma_{M_i}$  is subsequently used. This distribution is given as [22]

$$\bar{F}_{\sigma_{L_i} - \sigma_{M_i}}(t) = \int_{x=0}^{\infty} \int_{y=M_i}^{\infty} f_{\sigma_{M_i}, X_i(\sigma_{M_i})}(x, y) F_{\alpha_i t, \beta_i}(L_i - y) dy dx, \quad (1)$$

where  $F_{\alpha_i t, \beta_i}$  denotes the distribution function of a gamma process with parameters  $\alpha_i t$  and  $\beta_i$  and  $f_{\sigma_{M_i}, X_i(\sigma_{M_i})}(x, y)$  denotes the joint density function of  $(\sigma_{M_i}, X_i(\sigma_{M_i}))$  provided in [23].

After the just-in-time or opportunistic replacement of all  $m$  monitored components at the same maintenance time, the future evolution of the system does not depend any more on the past. Hence, maintenance times in which all the monitored-components are replaced are *regeneration points* for the process describing the evolution of the maintained system. However, describing the system state using renewal theory is rather tricky since many different corrective, preventive and just-in-time replacements of the components can occur between

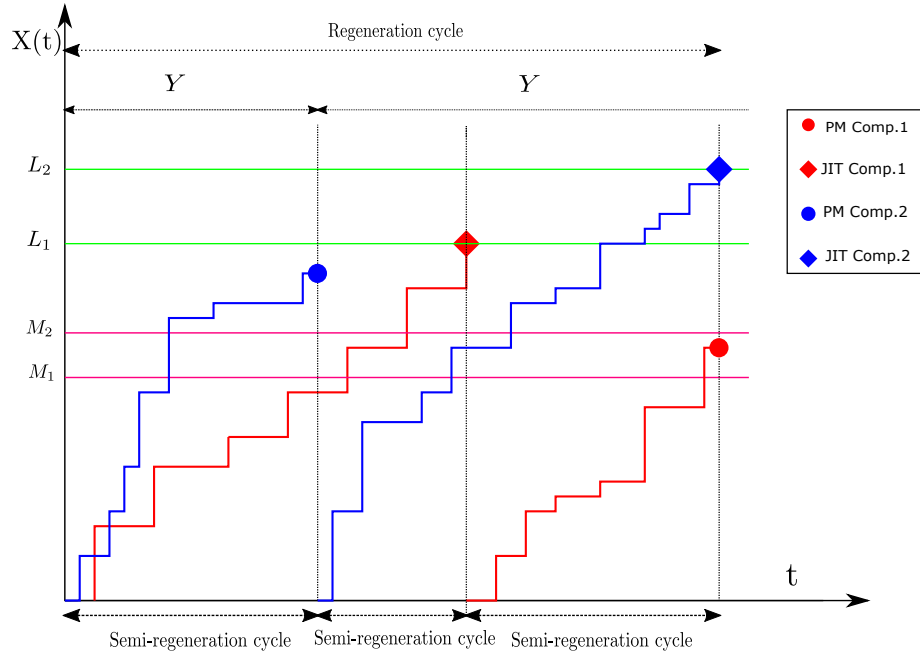


Figure 1: The maintenance policy for  $m = 2$  monitored components

two consecutive regeneration points. To deal with this problem, we can take advantage of the semi-regenerative properties of the process considering each *maintenance time* as a *semi-regeneration point*.

**Example 1.** Figure 1 shows the maintenance policy for  $m = 2$  monitored components with three maintenance cycles. The first maintenance action is a corrective action on a non-monitored component. At this opportunity, the degradation levels of components 1 and 2 are checked. The degradation level of component 2 exceeds the preventive threshold, so a preventive maintenance action is performed. The second maintenance action is just-in-time maintenance of component 1. At this opportunity, component 2 is left as is. The third maintenance action is just-in-time maintenance of component 2. At this opportunity, the degradation level of component 1 exceeds the preventive threshold, so it is maintained preventively, implying that this is a regeneration point.

### 3. Semi-regeneration process

Let  $T_1, T_2, \dots$ , be the maintenance times and let  $T_k^+$  be the instant of time just after the maintenance time  $T_k$ . A Markov chain with continuous state space  $[0, M_1) \times [0, M_2) \times \dots [0, M_m)$  is defined as

$$\{\mathbf{Z}_k = \mathbf{W}(T_k^+), \quad k = 1, 2, \dots\}, \quad (2)$$

where  $\mathbf{Z}_k$  is given by

$$\mathbf{Z}_k = \mathbf{W}(T_k^+) = (X_1(T_k^+), X_2(T_k^+), \dots, X_m(T_k^+)).$$

Due to the assumptions of the model (in particular, due to gamma process properties), the future evolution after  $T_k^+$  only depends on the system state at time  $T_k$ . It means that  $\{\mathbf{W}(t), t \geq 0\}$  with

$$W(t) = (X_1(t), X_2(t), \dots, X_m(t)),$$

is a semi-regenerative process with embedded Markov chain  $\{\mathbf{Z}_k\}$  and state space  $[0, M_1) \times [0, M_2) \times \dots [0, M_m)$ .

Starting with  $\mathbf{x} = (x_1, x_2, \dots, x_m)$ , where  $x_i < M_i$  for all  $i$ , the transition kernel of the embedded Markov chain given in Eq. (2) is

$$\mathbb{Q}_x(dy) = \mathbb{P}_x(\mathbf{W}(T_1^+) \in dy) = P(\mathbf{W}(T_1^+) \in dy \mid \mathbf{W}(0^+) = x). \quad (3)$$

To compute Eq. (3), the following cases after the first maintenance time  $T_1$  are considered:

1. All monitored components are replaced;
2. All monitored components are left as they are;
3. Some monitored components are replaced and the rest are left as they are.

Let  $\sigma_{M-x}$  and  $\sigma_{L-x}$  be the following vectors

$$\begin{aligned} \sigma_{M-x} &= (\sigma_{M_1-x_1}, \sigma_{M_2-x_2}, \dots, \sigma_{M_m-x_m}) \\ \sigma_{L-x} &= (\sigma_{L_1-x_1}, \sigma_{L_2-x_2}, \dots, \sigma_{L_m-x_m}) \end{aligned}$$

*Case 1. All monitored components are replaced.*

Given  $\mathbf{W}(0^+) = (x_1, x_2, \dots, x_m)$ , next maintenance is performed at time  $T_1 = \min(Y, \sigma_{\mathbf{L}-\mathbf{x}})$ . All the components are replaced in  $T_1$  if

$$\max(\sigma_{\mathbf{M}-\mathbf{x}}) \leq \min(Y, \sigma_{\mathbf{L}-\mathbf{x}}),$$

and this event has probability

$$\begin{aligned} \mathbb{Q}_x(dy_1, dy_2, \dots, dy_m) &= \prod_{i=1}^m \delta_0(dy_i) \int_0^\infty \dots \int_0^\infty \left( \prod_{i=1}^m f_{\sigma_{M_i-x_i}}(u_i) du_i \right. \\ &\quad \left. \int_{\max(u)}^\infty -\frac{d}{dw} \left( \bar{F}_Y(w) \prod_{i=1}^m \bar{F}_{\sigma_{L_i-x_i}-\sigma_{M_i-x_i}}(w-u_i) \right) dw \right), \end{aligned}$$

where  $\delta_0(dy_i)$  stands for the Dirac delta,  $\max(u) = \max(u_1, u_2, \dots, u_m)$  and  $\bar{F}_{\sigma_{L_i-x_i}-\sigma_{M_i-x_i}}$  is given by Eq. (1) replacing  $M_i$  by  $M_i - x_i$  and  $L_i$  by  $L_i - x_i$ .

*Case 2. All monitored components are left as they are.*

Given  $\mathbf{W}(0^+) = (x_1, x_2, \dots, x_m)$ , all the components are left as they are if  $Y < \min(\sigma_{\mathbf{M}-\mathbf{x}})$ . In this case, the expression for the kernel is

$$\mathbb{Q}_x(dy_1, dy_2, \dots, dy_m) = \int_0^\infty f_Y(v) \prod_{i=1}^m f_{\alpha_i v, \beta_i}(y_i - x_i) dy_i dv,$$

with  $(y_1, y_2, \dots, y_m) \in (0, M_1) \times (0, M_2) \times (0, M_m)$ .

*Case 3. Some monitored components are replaced and the rest are left as they are.*

Given  $\mathbf{W}(0^+) = (x_1, x_2, \dots, x_m)$ , some monitored components are replaced and the rest are left as they are if

$$\min(\sigma_{\mathbf{M}-\mathbf{x}}) \leq \min(Y, \sigma_{\mathbf{L}-\mathbf{x}}) \leq \max(\sigma_{\mathbf{M}-\mathbf{x}})$$

Let  $A$  ( $B$ ) the set of indexes of the replaced (non-replaced) components at the opportunity time

$$A = \{i \in I^m, \sigma_{M_i-x_i} \leq \min(Y, \sigma_{\mathbf{L}-\mathbf{x}})\}, \quad B = \{i \in I^m, \sigma_{M_i-x_i} > \min(Y, \sigma_{\mathbf{L}-\mathbf{x}})\}.$$

The computation of the kernel for this case is

$$\mathbb{Q}_x(dy_1, dy_2, \dots, dy_m) = \int_0^\infty \int_0^\infty \dots \int_0^\infty \prod_{s \in A} \delta_0(y_s) f_{\sigma_{M_s - x_s}}(u_s) du_s$$

$$\int_{\max(u)}^\infty \prod_{k \in B} f_{\alpha_k v, \beta_k}(y_k - x_k) dy_k dv \left( -\frac{d}{dv} \left( \bar{F}_Y(v) \prod_{s \in A} \bar{F}_{\sigma_{L_s - x_s} - \sigma_{M_s - x_s}}(v - u_s) \right) \right)$$

Hence, all the cases for  $\mathbb{Q}_x(dy_1, dy_2, \dots, dy_m)$  are contemplated.

#### 4. Numerical evaluation procedure

An optimal maintenance strategy is analyzed in this section. The long-run expected cost per unit time is chosen as objective cost function. It is given by

$$C_\infty = \lim_{t \rightarrow \infty} \frac{\mathbb{E}[C(t)]}{t}, \quad (4)$$

where  $C(t)$  denotes the cumulative cost in  $[0, t]$ . Denoting by  $R_1, R_2, \dots$  the times between regeneration points, using the renewal reward theorem we can approximate  $C_\infty$  focusing on the first cycle.

$$C_\infty = \frac{\mathbb{E}(C(R_1))}{\mathbb{E}(R_1)} \quad (5)$$

Since  $\{\mathbf{W}(t), t \geq 0\}$  is a semi-regenerative process with semi-regeneration times the maintenance times, the asymptotic cost given by (4) can be focused on a single semi-regenerative cycle defined as the time between two successive maintenance times.

Assuming that the Markov chain  $\{\mathbf{Z}_k, k = 1, 2, \dots\}$  comes back to the regeneration point  $(0, 0, \dots, 0)$  almost surely, it proves the existence of a vector  $\pi$  solution of the equation [11]

$$\pi(\cdot) = \int_0^{M_1} \int_0^{M_2} \dots \int_0^{M_m} \mathbb{Q}_x(\cdot) \pi(dx_1, dx_2, \dots, dx_m),$$

where  $\mathbb{Q}_x(\cdot)$  stands for the kernel of the Markov chain given by Eq. (3). Vector  $\pi$  is used to approximate the long-run maintenance cost rate given by Eq. (4) as

$$C_\infty = \frac{\mathbb{E}_\pi[C(T_1)]}{\mathbb{E}_\pi[T_1]}, \quad (6)$$

where  $T_1$  stands for the time to the first maintenance action.

Developing Eq. (6), we get that

$$C_\infty(M_1, M_2, \dots, M_m) = \frac{E_\pi[C^p(T_1)]}{E_\pi[T_1]} + \frac{E_\pi[C^j(T_1)]}{E_\pi[T_1]} + \frac{C^f E_\pi[N_f(T_1)]}{E_\pi[T_1]}, \quad (7)$$

where, given  $\pi$ ,  $E_\pi[C^p(T_1)]$ ,  $E_\pi[N^j(T_1)]$  and  $E_\pi[N^f(T_1)]$  stands for the expected cost due to preventive replacements, the expected cost due to just-in-time replacements and the expected number of sudden failures between two consecutive maintenance times respectively.

Terms  $E_\pi[C^p(T_1)]$ ,  $E_\pi[C^j(T_1)]$ ,  $E_\pi[N^f(T_1)]$  of the objective cost function given by Eq. (7) are next computed.

For fixed  $\mathbf{x} = (x_1, x_2, \dots, x_m)$ , the just-in-time replacement of the  $i$ -th component is performed if

$$\sigma_{L_i - x_i} \leq \min(Y, \sigma_{\mathbf{L} - \mathbf{x}}),$$

with expected cost

$$g_i(x_1, x_2, \dots, x_m) = C_i^j \int_0^\infty f_{\sigma_{L_i - x_i}}(u) \bar{F}_Y(u) \prod_{s=1, s \neq i}^m \bar{F}_{\sigma_{L_s - x_s}}(u) du.$$

Integrating by  $\pi$  and summing the expected cost of the just-in-time replacement of each component,

$$\begin{aligned} E_\pi[C^j(T_1)] &= \sum_{i=1}^m \int_0^{M_1} dx_1 \int_0^{M_2} dx_2 \dots \int_0^{M_m} \pi(\mathbf{x}) g_i(\mathbf{x}) dx_m \\ &= \int_0^{M_1} dx_1 \int_0^{M_2} dx_2 \dots \int_0^{M_m} \pi(\mathbf{x}) dx_m \int_0^\infty \frac{-d}{du} \left( \prod_{i=1}^m C_i^j \bar{F}_{\sigma_{L_i - x_i}}(u) \right) \bar{F}_Y(u) du. \end{aligned}$$

Between two consecutive maintenance times, the probability of a sudden failure is given by

$$E_\pi[N_f(T_1)] = \int_0^{M_1} dx_1 \int_0^{M_2} dx_2 \dots \int_0^{M_m} \pi(\mathbf{x}) dx_m \left( \int_0^\infty f_Y(v) \prod_{i=1}^m \bar{F}_{\sigma_{L_i - x_i}}(v) dv \right).$$

For the calculus of  $E_\pi[C^p(T_1)]$ , a preventive replacement is performed on the  $i$ -th monitored component in the next maintenance time if

$$\sigma_{M_i - x_i} < \min(\sigma_{\mathbf{L} - \mathbf{x}}, Y) < \sigma_{L_i - x_i},$$

with expected cost

$$h_i(x_1, x_2, \dots, x_m) = C_i^p \int_0^\infty f_{\sigma_{M_i-x_i}}(u) du \left( \int_u^\infty -\frac{d}{dw} \left( \bar{F}_Y(w) \prod_{s=1, s \neq i}^m \bar{F}_{\sigma_{L_s-x_s}}(w) \right) \bar{F}_{\sigma_{L_i-M_i}}(w-u) dw \right).$$

Integrating in  $\pi$  and summing for  $1 \leq i \leq m$ ,

$$E_\pi[C^p(T_1)] = \sum_{i=1}^m \int_0^{M_1} dx_1 \int_0^{M_2} dx_2 \dots \int_0^{M_m} \pi(\mathbf{x}) h_i(\mathbf{x}) dx_m.$$

Finally, the expected time between two consecutive maintenance times is given by

$$E_\pi[T_1] = \int_0^{M_1} dx_1 \int_0^{M_2} dx_2 \dots \int_0^{M_m} \pi(\mathbf{x}) \left( \int_0^\infty \bar{F}_Y(v) \prod_{i=1}^m \bar{F}_{\sigma_{L_i-x_i}}(v) dv \right) dx_m.$$

Hence, all the components of Eq. (refcostfunction) are analytically obtained.

The search of the optimal maintenance strategy

$$C(M_1^{opt}, M_2^{opt}, \dots, M_m^{opt}),$$

is reduced to the following optimization problem

$$C_\infty(M_1^{opt}, M_2^{opt}, \dots, M_m^{opt}) = \inf \{C_\infty(M_1, M_2, \dots, M_m)\}, \quad (8)$$

where  $C_\infty(M_1, M_2, \dots, M_m)$  is given by Eq. (7) with  $0 \leq M_i < L_i$  for all  $i$ .

## 5. Numerical experiments

The computation times using semi-regenerative techniques and renewal techniques are first compared in Section 5.1. A numerical example is given to illustrate how our results can be used to optimize a maintenance strategy in Section 5.2.

### 5.1. Comparison of computation times

A numerical example is given comparing the computation times of the expected cost rate using semi-regenerative techniques and using renewal techniques. This example has been performed using a computer with Intel (R)

Core (TM) i5-9600K @3.70GHZ with 16GB using a single node. Table 1 shows the computation times (in seconds) and the expected cost rate (in monetary units per time unit) for a system with  $m \in \{1, 2, \dots, 10\}$  monitored components where the expected cost rate using renewal techniques and semi-regenerative techniques is computed simulating Eq. (5) and Eq. (6), respectively.

Time between failures of the non-monitored components is modelled using an exponential distribution with parameter  $\lambda = 0.25$  failures per unit time. The following parameters for the gamma process and for the corresponding thresholds are used

$$\alpha_i = 1, \quad \beta_i = 1, \quad L_i = 7, \quad \text{and} \quad M_i = 6,$$

for  $i \in I^m$  and assuming the following costs (in monetary units)

$$C_i^j = 10, \quad C_i^p = 5, \quad \text{and} \quad C^f = 15.$$

For both techniques, 5,000 simulations have been performed to compute the expected cost rate. Table 1 shows that when the number of monitored components increases, the computation time using renewal techniques increases exponentially, while the computation time using semi-regenerative techniques increases linearly. Already for relatively small problem instances the difference becomes huge, and it is clear that real-life problem instances with hundreds, sometimes even thousands of components, can only be solved using semi-regenerative techniques.

### 5.2. Optimization of preventive maintenance thresholds

To illustrate our results, a system consisting of three monitored components is considered. The degradation of these monitored components evolves according to a homogeneous gamma process with parameters

$$\alpha_1 = 1, \quad \alpha_2 = 1.1, \quad \alpha_3 = 2, \quad \beta_1 = 1.01, \quad \beta_2 = 1.02, \quad \text{and} \quad \beta_3 = 2.$$

The just-in-time thresholds for these monitored components are given by

$$L_1 = 8, \quad L_2 = 8, \quad \text{and} \quad L_3 = 7.5.$$



	Semi-regenerative approach		Renewal approach	
$m$	$C_\infty$ (m.u/t.u)	Ex. Time (secs)	$C_\infty$ (m.u/t.u)	Ex. Time (secs)
2	5.3209	1.1160	5.3839	7.3752
3	6.0723	1.6600	6.0553	33.2089
4	6.6064	2.1808	6.7336	129.9082
5	7.4454	2.6982	7.3671	455.2273
6	7.9805	3.2244	7.9886	1,583.2
7	8.6020	3.7690	8.5955	4,770.7
8	9.2647	4.2817	9.1933	14,225
9	9.7952	4.8054	9.7812	66,818
10	10.4973	5.8387	10.3582	127,990

Table 1: Expected cost rates and computation times using semi-regenerative and renewal techniques

The system also contains non-monitored components. The time between failures of the non-monitored components is given by an exponential distribution with parameter  $\lambda = 0.25$  failures per unit time. The costs (in monetary units) for this system are given by

$$C_1^j = 1, \quad C_2^j = 2, \quad \text{and} \quad C_3^j = 2,$$

for the just-in-time maintenance tasks and

$$C_1^p = 1/3, \quad C_2^p = 2/3, \quad \text{and} \quad C_3^p = 1,$$

for the costs of preventive maintenance tasks. The cost of a corrective maintenance for the failures of the non-monitored component is equal to  $C^f = 5$  monetary units.

For  $M_1$  seven values are considered, equally spread on the interval  $(0, L_1 - 0.1]$ , for  $M_2$  six values are considered, equally spread on the interval  $(0, L_2 - 0.1]$ , and for  $M_3$  eight values are considered, equally spread on the interval  $(0, L_3 - 0.1]$ . This gives  $7 \cdot 6 \cdot 8 = 336$  points; at each of these points 10,000

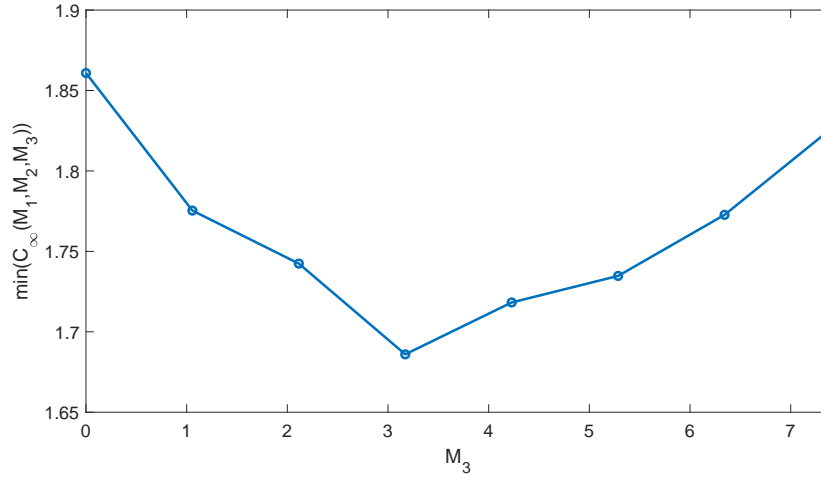


Figure 2: Optimal maintenance cost versus  $M_3$

simulations are performed with the semi-regenerative technique. Figure 2 shows the maintenance cost rate, given in Eq. (7), as a function of  $M_3$ . At each point, the values of  $M_1$  and  $M_2$  are used that give the lowest costs. The lowest maintenance cost is obtained for  $M_3 = 3.17$  and is  $C_\infty(M_1, M_2, 3.17) = 1.69$  monetary units per time unit. Figure 3 shows the maintenance cost rate as a function of  $M_1$  and  $M_2$  when  $M_3 = 3.17$ .”

## 6. Conclusions and further works

This paper deals with the problem of managing different maintenance actions for a system consisting of components under a condition-based maintenance policy and components under a corrective maintenance policy. An opportunistic maintenance policy is implemented: when a *just-in-time* maintenance action on a monitored component or when a corrective maintenance action is performed on a non-monitored component, the maintenance team takes this opportunity to simultaneously perform preventive maintenance on the monitored components whose degradation levels exceed a preventive threshold. For this complex system, an extremely fast procedure is developed to evaluate maintenance policies using semi-regenerative techniques. Numerical examples are given to show, first,

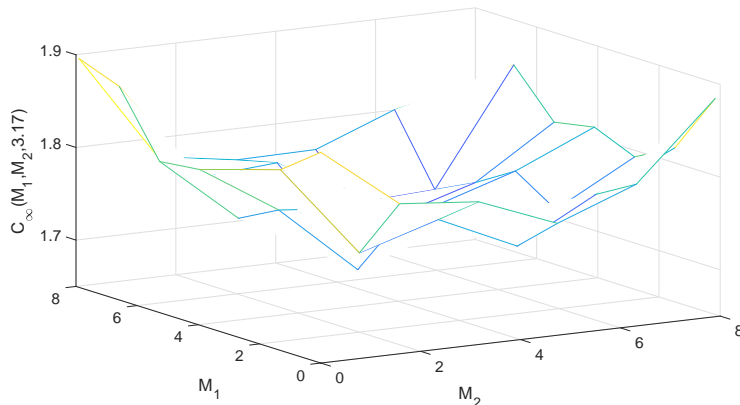


Figure 3: Optimal maintenance cost versus  $M_1$  and  $M_2$  for fixed  $M_3 = 3.17$

that our approach scales much better than approaches using commonly (renewal techniques) and, second, how our evaluation procedure can be used to optimize maintenance policies.

This paper is focused on an *evaluation* procedure, it would be interesting future research to come up with better *optimization* procedures. Furthermore, continuous monitoring is assumed in this paper, while there are also many situations in practice where monitoring happens at discrete moments only. It would be interesting to take this into account. Finally, adding components under usage-based or time-based maintenance would be very relevant too.

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