

# FROM KNOWLEDGE MODELS TO REASONING MODELS. EXEMPLIFICATION WITH A REASONING MODEL FOR SOLVING PROBLEMS OF ANALYTICAL REASONING

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## Abstract

Once instant access to information has been fully achieved through the Internet, the challenge ahead is to transform this information into knowledge. This requires selecting and hierarchizing the information conveniently, sequencing it, structuring it and interrelating it, so its meaning becomes apparent and new knowledge is generated. Concept maps are tools extensively used for this purpose, because they are a great help to perform these processes, existing software applications such as CmapTools that allow performing this task in an extraordinarily simple and versatile way. When an expert accomplished the work of transforming information into knowledge using a metacognitive activity, and expresses these results using multiple concept maps connected together, this way of relating the information in question in the form of knowledge can be offered to others as a model to follow and produces what is known as a Knowledge Model.

In this work we present a proposal for the extrapolation from the concept of Knowledge Model to the concept of Reasoning Model, i.e., linking together the pieces of reasoning applied by an expert to solve certain types of problems and offer to others the result of this exercise of "meta-reasoning" (reasoning about how to reason) as a proposal or model to follow.

To specify this proposal, a reasoning model has been made in the form of a concept map. In a structured way, several analytical reasoning problems are collected and related to each other depending on the type of reasoning used to find the solution.

By analytical reasoning problem we mean the problem that, once finished carefully reading and perfectly assimilating its statement, it produces a feeling of bewilderment, not knowing where to start looking for the solution (that is, the feeling of being in a problem). Also known as ingenious problems, we must contrive to find a way where to start. The solution of these problems does not require much academic knowledge, but a high IQ.

The reasoning model presented is accessible at the following web address: <http://grupoorion.unex.es:8001/servlet/SBReadResourceServlet?rid=1KGHCHK02-18X1B7K-2JTJ&partName=htmltext>. There we can find many links to problem statements of this kind, and the solutions personally found by the authors of this work. Also included is the result of the meta-reasoning carried out, in order to explain the way of reasoning that has led in each of the cases presented to find those solutions.

The experience has been completed with the use of this reasoning model by a group of students and teachers, and the evaluation of the results. This evaluation has shown that its use can contribute to help the users to learn how to reason.

Keywords: Knowledge models, reasoning models, problem solving, meta-reasoning.

## 1 INTRODUCTION: CONCEPT MAPS AND EXPERT MAPS

One of the most highlighted uses of concept maps involves capturing expert knowledge on a particular topic [1]. This use allows for the elaboration of a knowledge model that may be offered to less experienced individuals as an example of one possible way to choose, rank, relate, and structure concepts and relationships among the components that make up a given body of knowledge. While building knowledge is a personal task that each person must carry out on his or her own, this process

may benefit from the assistance of others. Such a form of collaborative learning allows others to propose relationships among concepts that each person may or may not adopt, depending on his or her own cognitive structure and on the compatibility of that structure with the new proposal. Accordingly, many studies have recently been conducted in this area, and increasing numbers of researchers are attempting to create knowledge models [2,3,4,5,6,7]

However, as learning remains conceptual with content such as facts, concepts, and principles, the proposal made by some authors to include phenomena as a new type of conceptual learning content should be extended [8]. Beyond the conceptual content, there are also procedural and attitudinal content, and in this study, we go a step further by addressing the following crucial and pressing questions. 1) What about reasoning? 2) Is it not prudent to consider reasoning as a type of learning content, which should be a fundamental objective of any professor when teaching his or her students? 3) Could “reasoning maps” be created in the same manner as conceptual pseudomaps? 4) Could such reasoning maps capture expert reasoning and represent it through conceptual pseudomaps? 5) Could reasoning models be created and offered as examples to others who are less experienced in the field? The creation and use of this type of map would provide a great tool for teaching students to reason, which demonstrates the great interest currently held in their development. These models would contain all types of learning content, including concepts, procedures, and reasoning. Therefore, we seek a term other than conceptual maps when referring to these new all-inclusive maps. Because experts on the topic in question create these maps, we propose calling them expert maps [8].

An expert map—as proposed here—is a type of concept map that contains all types of learning content created by an expert in a specific field; therefore, the map captures an expert’s knowledge, which is not limited to only conceptual knowledge but also includes procedural and reasoning knowledge. The term expert map refers to the type of map used to create knowledge models (of all types of knowledge, not only conceptual). These models can be defined as a collection of maps generated by capturing knowledge built by an expert in a specific field over a substantial period of time and after many hours of reflection on the cognitive structure after a given expert has engaged in meta-cognition and meta-reasoning. Figure 1 presents a concept map of an expert map integrated into conceptual, procedural, and reasoning. This map is available at the following web address: <http://grupoorion.unex.es:8001/servlet/SBReadResourceServlet?rid=1KSZ1LCSN-7X10PF-2SSN&partName=htmltext>

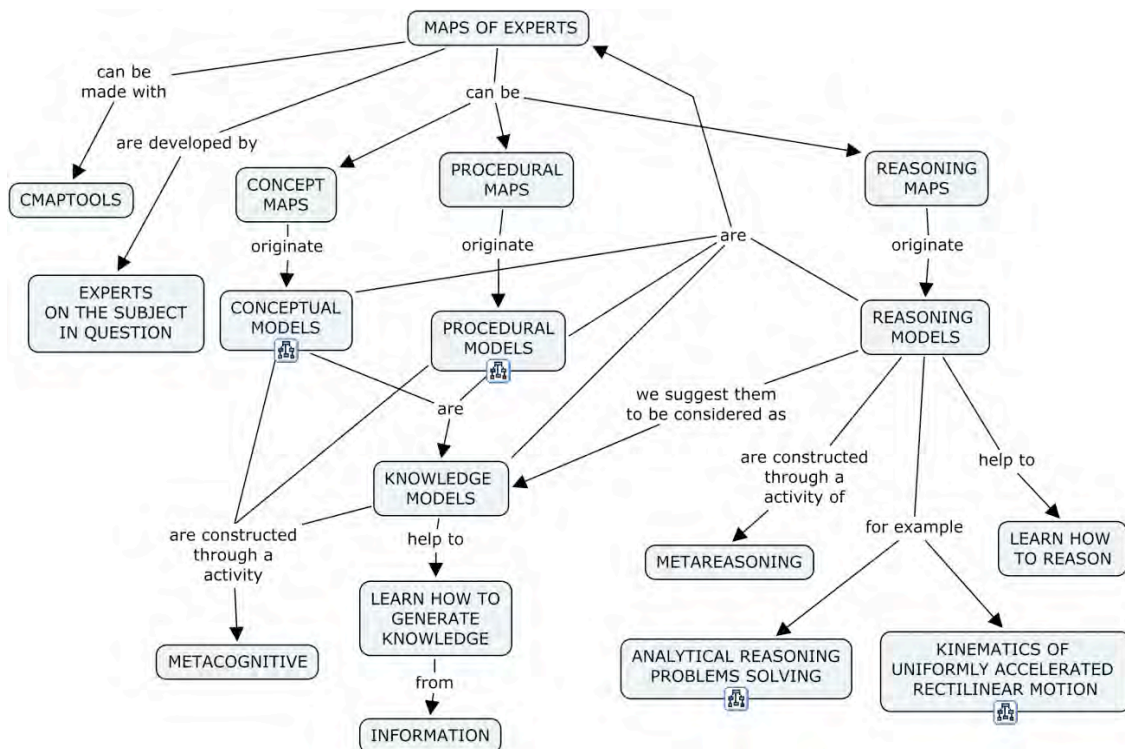


Fig. 1 Concept map of expert maps as an integration of conceptual, procedural, and reasoning models.

## **2 PROBLEM SOLVING: META-COGNITION AND META-REASONING**

As problem solving may be considered one of the didactic foundations of any scientific discipline, a great number of researchers have conducted studies on problem solving strategies in the sciences [9, 10, 11, 12, 13, 14, 15]. For example, Solaz-Portolés and Sanjosé, [16] analyzed different cognitive variables that influence problem solving. Other researchers [17], have also noted that it is necessary to conduct an adequate didactic approach toward cognitive and meta-cognitive abilities to learn how to learn. Furthermore, Gök and Silay [18] determined that students instructed in meta-cognitive strategies for problem solving obtain better results when solving problems.

The term meta-cognition, or meta-cognitive knowledge, has been defined as the knowledge one has about the factors affecting cognitive activities, that is, knowing how one acquires knowledge [19, 20]. Many researchers have noted two interrelated components of meta-cognition: knowledge and regulation of cognition [10]. Extant literature further notes that meta-cognition requires knowing what one wants to obtain (objectives) and how to obtain it (self-regulation or strategy). Accordingly, this study posits that a student is cognitively mature when he or she knows what comprehension is and how he or she cognitively comprehends [21].

This concept of meta-cognition is fundamental to problem solving [22]. The development of these skills helps a student to form mental models of a problem and to choose the best strategy for solving it. In other studies [23], it was concluded that students who obtain the best results in solving electricity problems are those who formed a mental map of the electromagnetic field. This mental map is similar to a map that an expert (someone with extensive knowledge in a particular field of study) would build. Concretely, these students built concept maps [1] that comprise differentiated, related, and ranked concepts. D.P. Simon and H.A. Simon [24] showed that there are differences between problem-solving strategies used by experts and by novices. Other studies [25] noted that experts use diagrams containing the information most relevant to the solution when considering problems. In addition, it has been demonstrated [26] that experts at solving physics problems are those who conduct an exhaustive and qualitative analysis of the problem and reflect on it using a planning and control scheme. Such meta-cognitive skills engender success in problem solving [27].

Consistent with the above precedents, this study has evolved from the concept of meta-cognition to that of meta-reasoning. The term meta-reasoning can be defined as reasoning about the reasoning process and may explain the way one seeks reasoning to solve a particular problem. When meta-reasoning is conducted by an expert to solve certain types of problem, the result may be offered as an example to other individuals less experienced in the task. Thus, such an endeavor may constitute a reasoning model. To capture this expert meta-reasoning in a structured and organized manner, concept maps have been used as a basic didactic tool. However, because these maps comprise reasoning procedures, they are termed reasoning maps here.

## **3 METHODOLOGY: ELABORATION OF A REASONING MODEL FOR ANALYTICAL REASONING PROBLEM SOLVING**

As an example of a reasoning model we present in Figure 2 an expert map, which captures the way of reasoning of an expert teacher in solving analytical reasoning problems. The model is available on our Cmap Website “Universidad de Extremadura (España)” in the directory “Mapas de Experto” where they can be used interactively through the CmapTools application. (It can also be viewed using your web browser at the address: <http://grupoorion.unex.es:8001> in the folder: “Mapa de experto de modelo de razonamiento analítico”

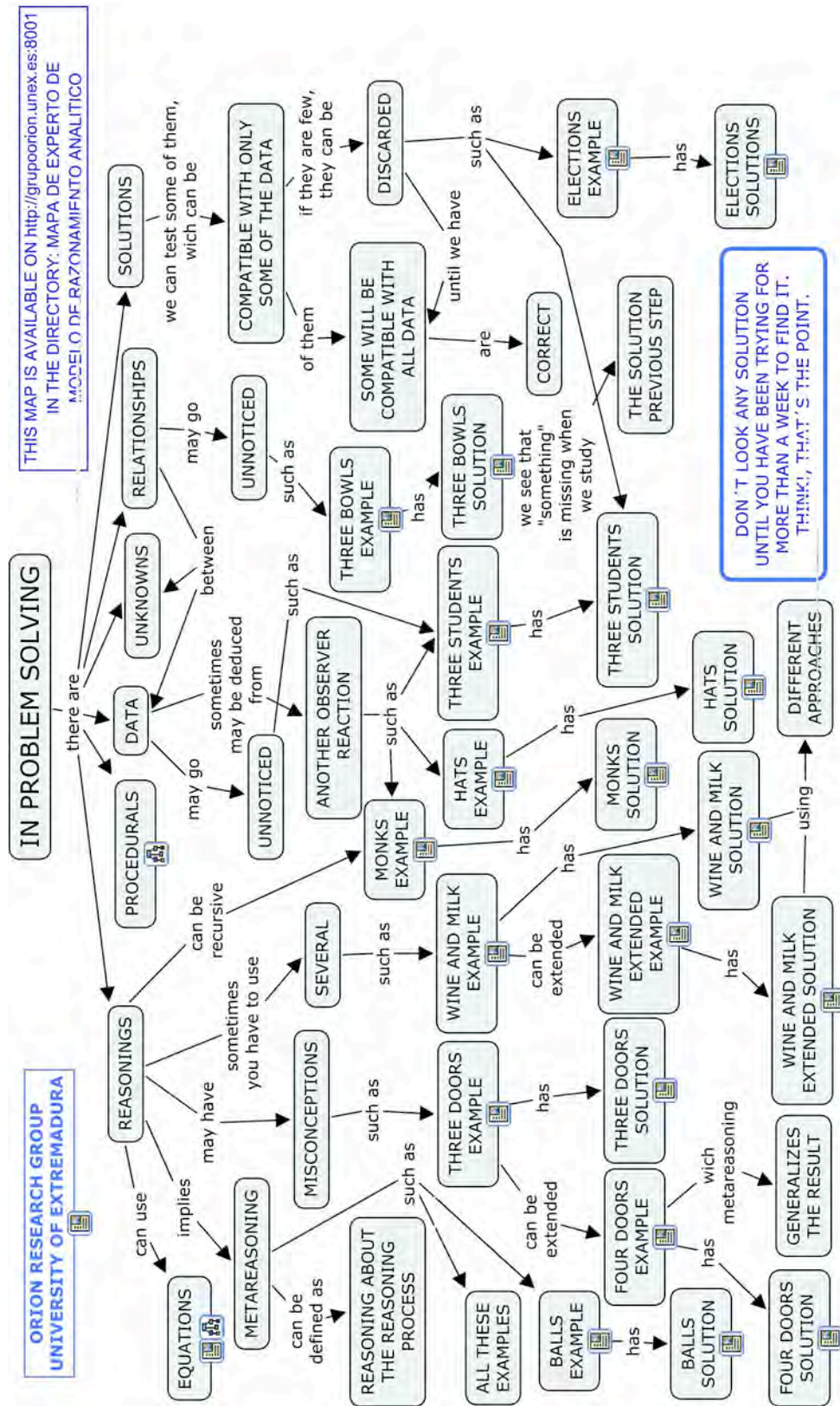


Fig. 2 Reasoning model which captures the way of reasoning of an expert teacher in to solving problems of analytical reasoning



When trying to solve a problem it can happen that, once we finish reading the statement, we know how to proceed. On the other hand, sometimes we are completely clueless about where to start. In the first case we are not facing a real problem; it would be more of a problem-solving exercise which workaround is known beforehand. In the second case we can choose to implement a known solution search procedure, or start reasoning how to find it. In either of these cases is essential to carefully read and reread the problem statement looking for all the existing necessary data that will be used to find the solution.

In most of the "difficult" problems, it is hard to extract data from the statement (see the example of "the three students" on the reasoning model of Figure 2). We must also pay attention to the relationships that can be established between the data and the unknowns. They pass often unnoticed, making impossible to find the solution (see the example of the "three bowls" on the reasoning model). Another preliminary issue to be considered is the possible existence of misconceptions about what is stated in the statement. We may perceive the situation described in a wrong way, but seemingly so obvious to us that will make it impossible to think about other possible options (including the right one) (see the example of the "three doors" on the model of reasoning).

If we take into account these three starting points (extract all the pieces of data from the statement, consider all possible relationships between them and the unknowns, and carefully avoid any misconception), and we are not really in the mood for think, we can apply some previously set solution search procedure. We may choose among the following:

- 1) If the relations between the pieces of data and the unknowns can be formulated as equations and we can set a number of equations equal to the number of unknowns, we can solve the resulting equation system and find the solution (see example in the kinematic problem solving model on the model of reasoning).
- 2) "The crude statement methodology", which consists on trying with different solutions attempts until we find one that meets the provisions of the statement. This procedure, besides inelegant, may be impossible to implement in practice. There may be very numerous possible solutions, and it is very unlikely that we get the right one within a reasonable amount of time.
- 3) "Trial and error methodology", where we follow the path set out in the statement, stopping in each step to check or reject the option until we get it right. This procedure (just a little more elegant than the previous) may also be impossible to apply in cases where the possible options are very numerous.

In these two latter cases, we can use the aid of a computer, which will apply a "brute force" routine. We will not study these options, as they are considered inelegant.

- 4) Take into account some of the data that limit the possible solutions, and discard those that are not consistent with other data, until we get to the right solution compatible with all data. This procedure derives from "the crude statement", but provides a simplification of the level of difficulty. As it requires a series of reasoning, it significantly enhances its elegance. That is, the methodology is part procedure and part reasoning, and thus can be considered in both classifications.

Figure 3 shows a conceptual map in which there is specified some procedures for semi-automatic problem solving. This page can be accessed following the link of the concept "PROCEDURES" on the reasoning model of Figure 2.

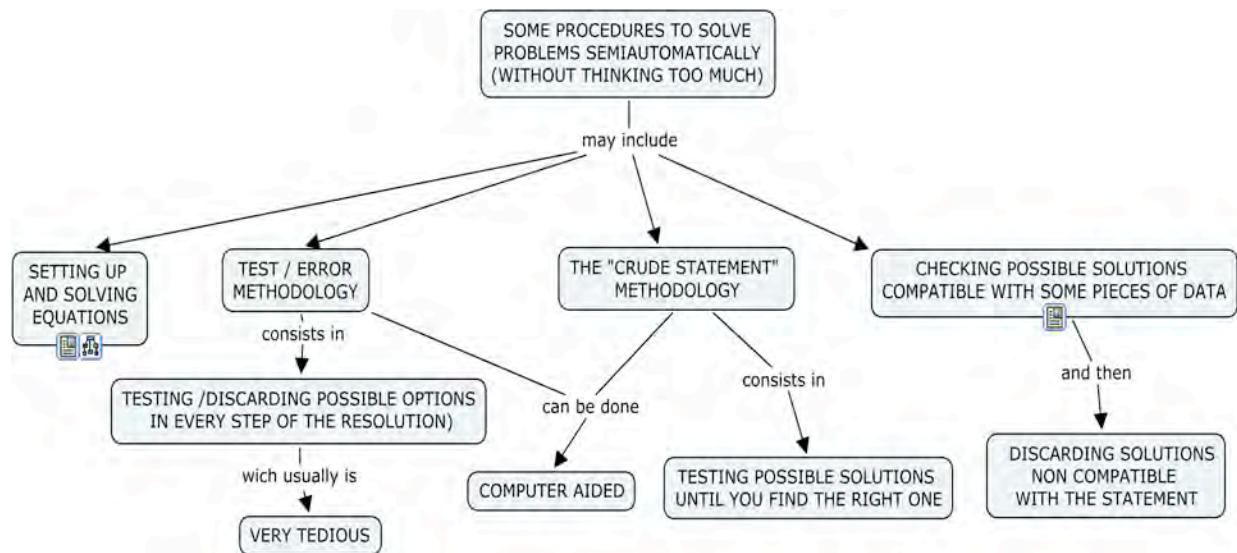


Fig. 3 Concept map with different methods of problem solving.

If the attempts to solve the problem in question using the procedures described above are unsuccessful (or if we just want to think), we can perform different analytical reasonings that could help us find the solution (as those shown in the reasoning model of Figure 2). Some of them are:

- 1) When the problem has a high degree of difficulty, it is usually due because we do not consider all the data needed to get to the solution. In addition carefully to re-read the statement looking for any data that may have passed unnoticed, it is good practice to try to find useful information by analyzing the problem from a different perspective (see the hats example on the reasoning model).
- 2) Sometimes you have to use a recursive reasoning (see the monks example on the reasoning model, where we get its solution by recursively applying the reasoning that leads to the solution to the hats problem).
- 3) Sometimes it's some of the relationships between the data and the unknowns which has gone unnoticed. This absence can be detected by studying the different steps of the solution of the problem. If we stop to analyze the step immediately prior to the solution and we find that it is impossible to proceed, we must conclude that "something is missing" (see the three bowls example on the reasoning model) .
- 4) Sometimes, the reasoning that has allowed us to find the solution of a given problem has to be abandoned, and replaced by one completely different when the problem is generalized (see the wine and milk example on the reasoning model). In this case, if we insist on continue with the initial reasoning we will just waste time.
- 5) As we have already discussed on the procedures, sometimes you can make a reasoning, which permits a limited number of possible solutions compatible with one of the pieces of data. Then, we will discard those that are not compatible with the rest of the data until we get to the solution (see the elections example on the reasoning model).

In all cases it is very convenient to proceed with a meta-reasoning process (reason about how we are reasoning), and solve extensions to the problem in question and its generalization. This often lead to a full understanding of the mechanism of resolution of the problem, and to the learning of useful types of reasoning which can be applied in the future to solve similar problems (see the four-doors example on the reasoning model). This in-depth meta-reasoning process carried out by the expert teacher is what originates the reasoning model that helps our students to simplify notably the difficulty of the problems that they may find: it teaches them to reason when faced with solving problem.

## 4 CONCLUSIONS

Meta-reasoning helps our students to decrease the difficulty of the problems involved. Specifically, the creation and use of these Expert Maps are a great aid for "teaching how to think" to our students. This shows the great interest that presents the development of such maps, in science subjects above all.

As concepts, also the reasoning used in problem solving can be set up in a hierarchy, interrelated and structured conforming Reasoning Maps.

Through a deep meta-reasoning activity that makes us aware of the reasonings used, we can build Reasoning Models as various conveniently related Reasoning. These Reasoning Models may be offered to people less experienced than their authors, as a proposal to follow in order to learn how to find answers to those problems. In our teaching practice, we have found that the Reasoning Models are a powerful tool to help people to "learn to reason" and to seek the appropriate reasonings that may allow them to find the solution to the problems posed.

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