Supplemental Information for "Numerical construction of the Aizenman-Wehr metastate"

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This note illustrates the lack of measurable size-corrections on the computation of the ζ exponent reported on the main text.

As explained in the main text, once the system size L is effectivily taken to infinity, the MAS susceptibility $\chi_\rho(W,R)$ still depends on the window sizes R and W. The scaling relation

$$\chi_\rho(W, R\to\infty) \propto W^\zeta \qquad (1)$$

defines the ζ exponent, but it is inconvenient for numerical work (where both W and R are finite). Fortunately, finite-size scaling solves this problem [1].

Indeed, we expect for finite R and W (see main text), a scaling behaviour

$$\chi_\rho(W,R) = R^\zeta f(W/R)\,. \qquad (2)$$

Consistency with Eq. (1) is obtained if the scaling function $f(x = W/R)$ scales in the limit $x \to 0$ as a power law $f(x) \sim x^\zeta$.

Eq. (2) is expected to be exact only in the limit of large W and R [1], hence one needs to check for size corrections. We do so with the quotients method [2–4], which produces *effective* ζ estimates at a well defined lengthscale. The size dependence can be assessed later on. Specifically, take two sizes-pairs (W_1, R_1), (W_2, R_2) with the same value of W/R, which ensures the cancellation of scaling functions in the quotient

$$\frac{\chi_\rho(W_2 = xR_2, R_2)}{\chi_\rho(W_1 = xR_1, R_1)} = \left(\frac{W_2 f(x)}{W_1 f(x)}\right)^\zeta = \left(\frac{W_2}{W_1}\right)^\zeta\,. \qquad (3)$$

W/R	L/R	(W_1, W_2)	$\zeta^{\rm eff}$
1/2	2	(4,6)	2.18(40)
2/3	2	(4,8)	2.59(22)
1	2	(8,12)	2.37(26)
		(6,8)	2.14(37)
		(6,12)	2.28(18)

TABLE I. The effective ζ exponent, Eq. (3), depends on the two lengths W_1 and W_2 and on the ratio $W_1/R_1 = W_2/R_2$.

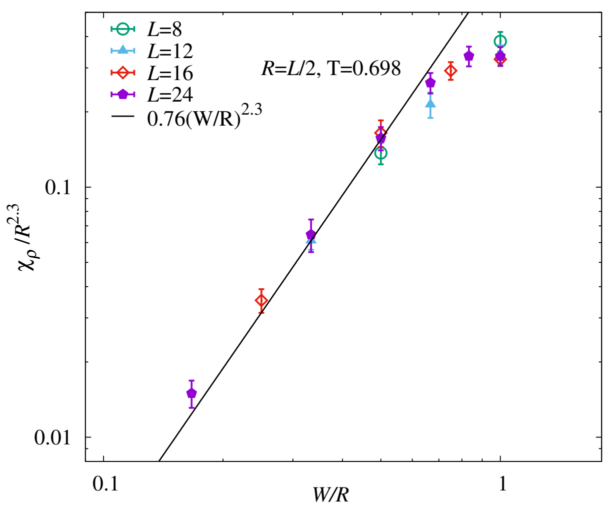

FIG. 1. Ilustration of the scaling in Eq. (2) (data from main text). At small values of the scaling variable $x = W/R$ the scaling function behaves as a power law $f(x) \propto x^\zeta$.

The resulting determination of ζ, see Table I, is fully compatible with the main-text result $\zeta = 2.3(3)$. Furthermore, no significant size-dependence emerges from Table I. Besides, a power law with the main-text ζ estimation interpolates the data nicely in the region of $x < 0.75$, see Fig. 1.

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